Lecture 2: Optimal Income Taxation

Stefanie Stantcheva

Fall 2016

GOALS OF THIS LECTURE

1) Take a look at **actual taxes and transfers** in the U.S.

2) Understand the **core optimal income tax model**: linear and nonlinear taxes in the Saez (2001) framework.

General method, intuitive, sufficient statistics.

3) Introduce the mechanism design approach of **Mirrlees (1971)**.

Incentive compatibility, optimal control.

TAXATION AND REDISTRIBUTION

Key question: Should government reduce inequality using taxes and transfers?

1) Governments use **taxes** to raise revenue

2) This revenue funds **transfer** programs:

a) Universal Transfers: Education, Health Care (only 65+ in the US), Retirement and Disability

b) Means-tested Transfers: In-kind (e.g., public housing, nutrition, Medicaid in the US) and cash

Modern governments raise large fraction of GDP in taxes (30-45%) and spend significant fraction of GDP on transfers

This lecture follows Piketty and Saez '13 **handbook chapter**

GOAL: TAKE A LOOK AT ACTUAL TAX SYSTEM

Sometimes you are an optimal tax theorist and don't know the actual top tax rates – it's weird.

You need to know institutional details. It's not boring. It's crucial.

You should not try to capture all institutional details in your models. But unless you know them, you cannot argue they are second-order. (Sometimes the devil is in the detail, sometimes not).

The tax system reflects

i) social judgements made by people and policy makers and

ii) lobbying, political economy, interest groups.

Understand the implicit social judgements behind the tax system.

Question them! Which constraints are truly "irremovable"?

FACTS ON US TAXES AND TRANSFERS

References: Comprehensive description in Gruber undergrad textbook (taxes/transfers) and Slemrod-Bakija (taxes)

http://www.taxpolicycenter.org/taxfacts/

A) Taxes: (1) individual income tax (fed+state), (2) payroll taxes on earnings (fed, funds Social Security+Medicare), (3) corporate income tax (fed+state), (4) sales taxes (state)+excise taxes (state+fed), (5) property taxes (state)

B) Means-tested Transfers: (1) refundable tax credits (fed), (2) in-kind transfers (fed+state): Medicaid, public housing, nutrition (SNAP), education (3) cash welfare: TANF for single parents (fed+state), SSI for old/disabled (fed)

FEDERAL US INCOME TAX

US income tax assessed on **annual family** income (not individual) [most other OECD countries have shifted to individual assessment]

Sum all cash income sources from family members (both from labor and capital income sources) = called **Adjusted Gross Income (AGI)**

Main exclusions: fringe benefits (health insurance, pension contributions), imputed rent of homeowners, interest from state+local bonds, unrealized capital gains

FEDERAL US INCOME TAX

Taxable income = AGI - personal exemptions - deduction

personal exemptions = $4K * # family members (in 2016)

deduction is max of standard deduction or itemized deductions

Standard deduction is a fixed amount depending on family structure ($12.6K for couple, $6.3K for single in 2016)

Itemized deductions: (a) state and local taxes paid, (b) mortgage interest payments, (c) charitable giving, various small other items

[about 10% of AGI lost through itemized deductions, called tax expenditures]

FEDERAL US INCOME TAX: TAX BRACKETS

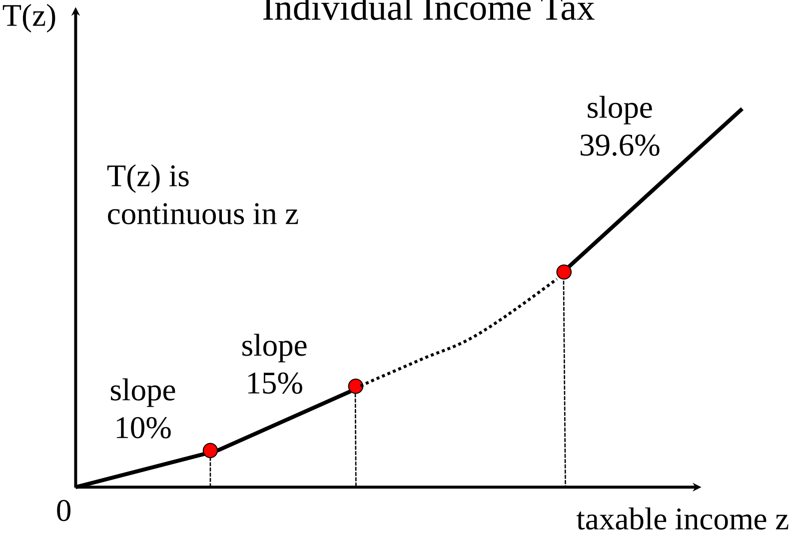
Tax $T(z)$ is piecewise linear and continuous function of taxable income z with constant marginal tax rates (MTR) $T'(z)$ by brackets

In 2013+, 7 brackets with MTR 10%,15%,25%,28%,33%,35%, 39.6% (top bracket for z above $470K), indexed on price inflation

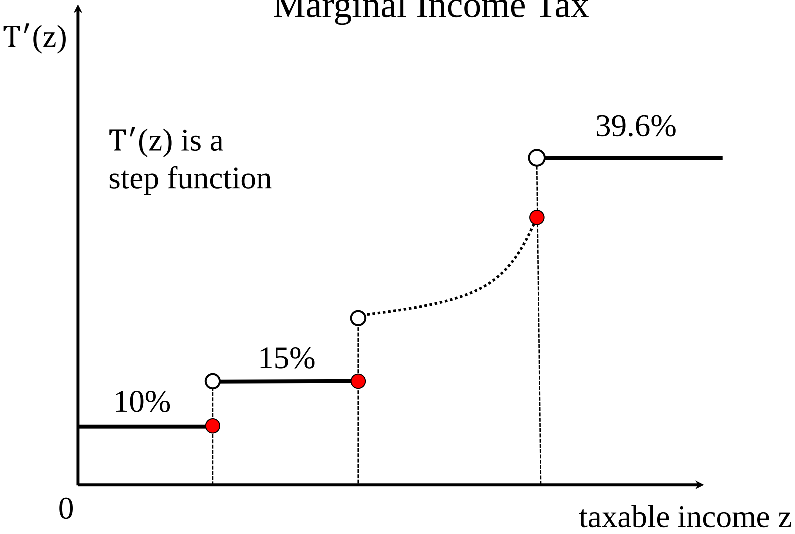
Lower preferential rates (up to a max of 20%) apply to dividends (since 2003) and realized capital gains [in part to offset double taxation of corporate profits]

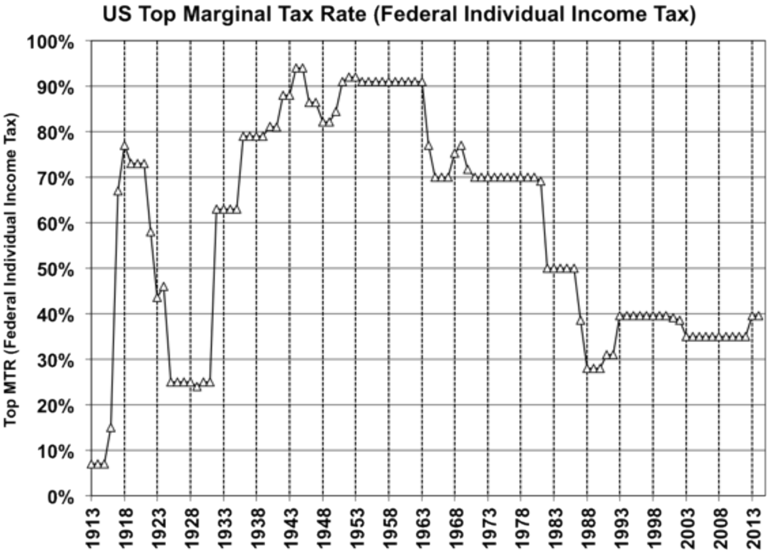
Tax rates change frequently over time. Top MTRs have declined drastically since 1960s (as in many OECD countries)

Individual Income Tax



Marginal Income Tax





Source: IRS, Statistics of Income Division, Historical Table 23

In practice...

Form **1040** U.S. Individual Income Tax Return **2010** (99) IRS Use Only - Do not write or staple in this space.

Name, Address, and SSN

PRINT CLEARLY

For the year Jan. 1-Dec. 31, 2010, or other tax year beginning , 2010, ending , 20 | OMB No. 1545-0074

Your first name and initial	Last name	Your social security number
BARACK H.	OBAMA	
If a joint return, spouse's first name and initial	Last name	Spouse's social security number
MICHELLE L.	OBAMA	
Home address (number and street). If you have a P.O. box, see instructions.	Apt. no.	Make sure the SSN(s) above ▲ and on line 6c are correct.
1600 PENNSYLVANIA AVENUE, NW		
City, town or post office, state, and ZIP code.		Checking a box below will not change your tax or refund.
WASHINGTON, DC 20500		

See separate instructions.

Presidential Election Campaign ▶ Check here if you, or your spouse if filing jointly, want $3 to go to this fund ▶ [X] You [X] Spouse

Filing Status

Check only one box.

1 [ ] Single
2 [X] Married filing jointly (even if only one had income)
3 [ ] Married filing separately. Enter spouse's SSN above and full name here. ▶
4 [ ] Head of household (with qualifying person). If the qualifying person is a child but not your dependent, enter this child's name here. ▶
5 [ ] Qualifying widow(er) with dependent child

Exemptions

6a [X] **Yourself.** If someone can claim you as a dependent, **do not** check box 6a
b [X] **Spouse**

Boxes checked on 6a and 6b: 2

c Dependents:

(1) First name Last name	(2) Dependent's social security number	(3) Dependent's relationship to you	(4) ✓ if child under age 17 qualifying for child tax credit
MALIA A OBAMA		DAUGHTER	X
NATASHA M OBAMA		DAUGHTER	X

No. of children on 6c who:
• lived with you: 2
• did not live with you due to divorce or separation (see instructions):

Dependents on 6c not entered above:

If more than four dependents, see instructions and check here ▶ [ ]

d Total number of exemptions claimed .... Add numbers on lines above ▶ 4

Income

Attach Form(s) W-2 here. Also attach Forms W-2G and 1099-R if tax was withheld.

If you did not get a W-2,

7	Wages, salaries, tips, etc. Attach Form(s) W-2	7	395,188.
8a	Taxable interest. Attach Schedule B if required	8a	8,066.
b	Tax-exempt interest. Do not include on line 8a — 8b		
9a	Ordinary dividends. Attach Schedule B if required	9a	9,997.
b	Qualified dividends — 9b 2,159.		
10	Taxable refunds, credits, or offsets of state and local income taxes STMT 2 STMT 3	10	1,151.
11	Alimony received	11	
12	Business income or (loss). Attach Schedule C or C-EZ	12	1,382,889.
13	Capital gain or (loss). Attach Schedule D if required. If not required, check here ▶ [ ]	13	-3,000.

Single:

Taxable Income	Tax Rate
$0 to $9,225	10%
$9,226 to $37,450	$922.50 plus 15% of the amount over $9,225
$37,451 to $90,750	$5,156.25 plus 25% of the amount over $37,450
$90,751 to $189,300	$18,481.25 plus 28% of the amount over $90,750
$189,301 to $411,500	$46,075.25 plus 33% of the amount over $189,300
$411,501 to $413,200	$119,401.25 plus 35% of the amount over $411,500
$413,201 or more	$119,996.25 plus 39.6% of the amount over $413,200

Married Filing Jointly or Qualifying Widow(er):

Taxable Income	Tax Rate
$0 to $18,450	10%
$18,451 to $74,900	$1,845.00 plus 15% of the amount over $18,450
$74,901 to $151,200	$10,312.50 plus 25% of the amount over $74,900
$151,201 to $230,450	$29,387.50 plus 28% of the amount over $151,200
$230,451 to $411,500	$51,577.50 plus 33% of the amount over $230,450
$411,501 to $464,850	$111,324.00 plus 35% of the amount over $411,500
$464,851 or more	$129,996.50 plus 39.6% of the amount over $464,850

Married Filing Separately:

Taxable Income	Tax Rate
$0 to $9,225	10%
$9,226 to $37,450	$922.50 plus 15% of the amount over $9,225
$37,451 to $75,600	$5,156.25 plus 25% of the amount over $37,450
$75,601 to $115,225	$14,693.75 plus 28% of the amount over $75,600
$115,226 to $205,750	$25,788.75 plus 33% of the amount over $115,225
$205,751 to $232,425	$55,662.00 plus 35% of the amount over $205,750
$232,426 or more	$64,998.25 plus 39.6% of the amount over $232,425

FEDERAL US INCOME TAX: AMT AND CREDITS

Alternative minimum tax (AMT) is a parallel tax system (quasi flat tax at 28%) with fewer deductions: actual tax $= \max(T(z), AMT)$ (hits 2-3% of tax filers in upper middle class)

Tax credits: Additional reduction in taxes

(1) **Non refundable** (cannot reduce taxes below zero): foreign tax credit, child care expenses, education credits, energy credits

(2) **Refundable** (can reduce taxes below zero, i.e., be net transfers): EITC (earned income tax credit, up to \$3.4K, \$5.6K, \$6.3K for working families with 1, 2, 3+ kids), Child Tax Credit (\$1K per kid, partly refundable)

FEDERAL US INCOME TAX: TAX FILING

Taxes on year t earnings are withheld on paychecks during year t (pay-as-you-earn) (Why?)

Income tax return filed in Feb-April 15, year $t+1$ [filers use either software or tax preparers, **huge** private industry]

Most tax filers get a tax refund as withholdings $>$ net taxes owed

Payers (employers, banks, etc.) send income information to govt (3rd party reporting)

Information + withholding at source is key for successful enforcement

MAIN MEANS-TESTED TRANSFER PROGRAMS

1) **Traditional transfers:** managed by welfare agencies, paid on monthly basis, high stigma and take-up costs $\Rightarrow$ low take-up rates

Main programs: Medicaid (health insurance for low incomes), Supplemental Nutritional Assistance Program (SNAP, former food stamps), public housing, Temporary Assistance to Needy Families (TANF, traditional welfare), Supplemental Security Income (aged+disabled)

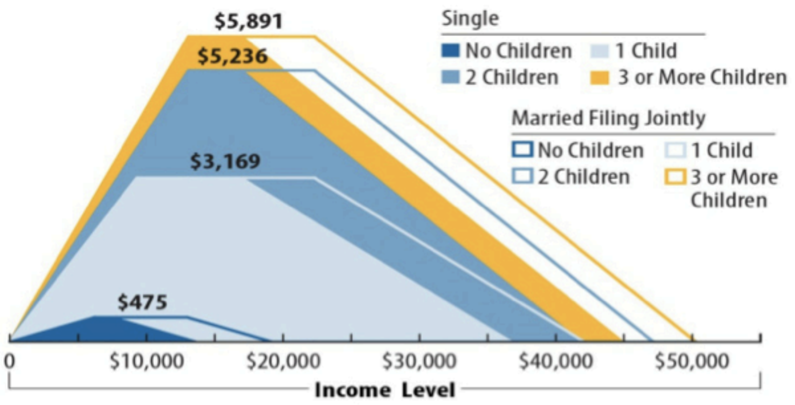
2) **Refundable income tax credits:** managed by tax administration, paid as an annual lumpsum in year $t+1$, low stigma and take-up cost $\Rightarrow$ high take-up rates

Main programs: EITC and Child Tax Credit [large expansion since the 1990s] for low income working families with children

$\rightarrow$ move has been from "support the very poor" to "support working low-income."

Figure 1

EITC refunds by family size and income (CBPP 2013)



Source: Center on Budget and Policy Priorities.

BOTTOM LINE ON ACTUAL TAXES/TRANSFERS

1) Based on current income, family situation, and disability (retirement) status $\Rightarrow$ Strong link with **current ability to pay**

2) Some allowances made to reward / encourage certain behaviors: charitable giving, home ownership, savings, energy conservation, and more recently work (refundable tax credits such as EITC)

Do you think this is the role of the tax system?

3) Provisions pile up overtime making tax/transfer system more and more complex until significant simplifying reform happens (such as US Tax Reform Act of 1986)

Sometimes such simplifications don't happen $\rightarrow$ e.g.: Europe (France). Motto: any vested interest you create will be impossible to remove.

KEY CONCEPTS FOR TAXES/TRANSFERS

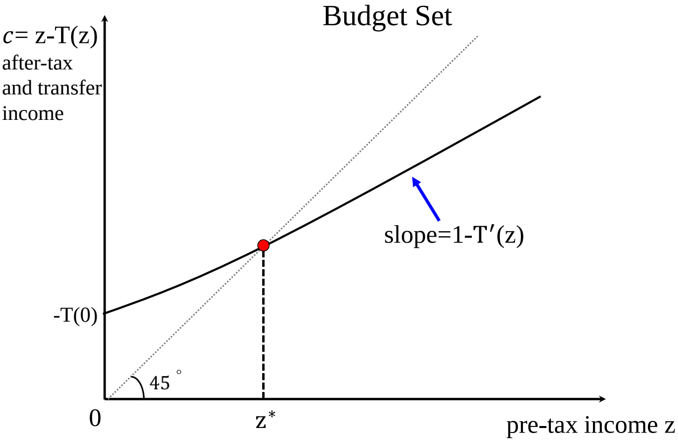
1) Transfer benefit with zero earnings $-T(0)$ [sometimes called demogrant or lumpsum grant]

2) Marginal tax rate (or phasing-out rate) $T'(z)$: individual keeps $1-T'(z)$ for an additional $1 of earnings (intensive labor supply response)

3) Participation tax rate $\tau_p = [T(z) - T(0)]/z$: individual keeps fraction $1-\tau_p$ of earnings when moving from zero earnings to earnings z (extensive labor supply response):

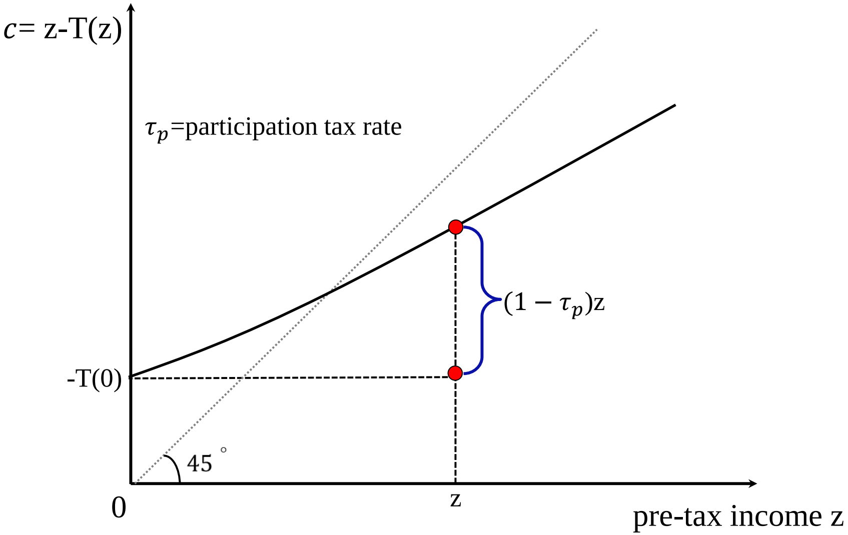
$$z - T(z) = -T(0) + z - [T(z) - T(0)] = -T(0) + z \cdot (1 - \tau_p)$$

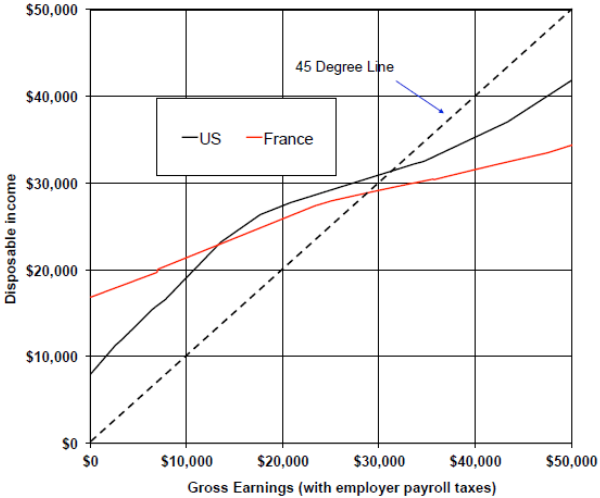
4) Break-even earnings point z^*: point at which $T(z^*) = 0$



If line is steeper is that more or less redistribution?

What is perfect redistribution? What is no redistribution?


c= z-T(z)
τ_p=participation tax rate
$(1-\tau_p)$z
-T(0)
45 °
0
z
pre-tax income z



Source: Piketty, Thomas, and Emmanuel Saez (2012)

OPTIMAL TAXATION: SIMPLE MODEL WITH NO BEHAVIORAL RESPONSES

Utility $u(c)$ strictly increasing and concave

Same for everybody where c is after tax income.

Income is z and is fixed for each individual, $c = z - T(z)$ where $T(z)$ is tax on z. z has density distribution $h(z)$

Government maximizes **Utilitarian** objective:

$$\int_0^\infty u(z - T(z))h(z)dz$$

subject to **budget constraint** $\int T(z)h(z)dz \geq E$ (multiplier λ)

SIMPLE MODEL WITH NO BEHAVIORAL RESPONSES

Form lagrangian: $L = [u(z - T(z)) + \lambda \cdot T(z)] \cdot h(z)$

First order condition (FOC) in $T(z)$:

$$0 = \frac{\partial L}{\partial T(z)} = [-u'(z - T(z)) + \lambda] \cdot h(z) \Rightarrow u'(z - T(z)) = \lambda$$

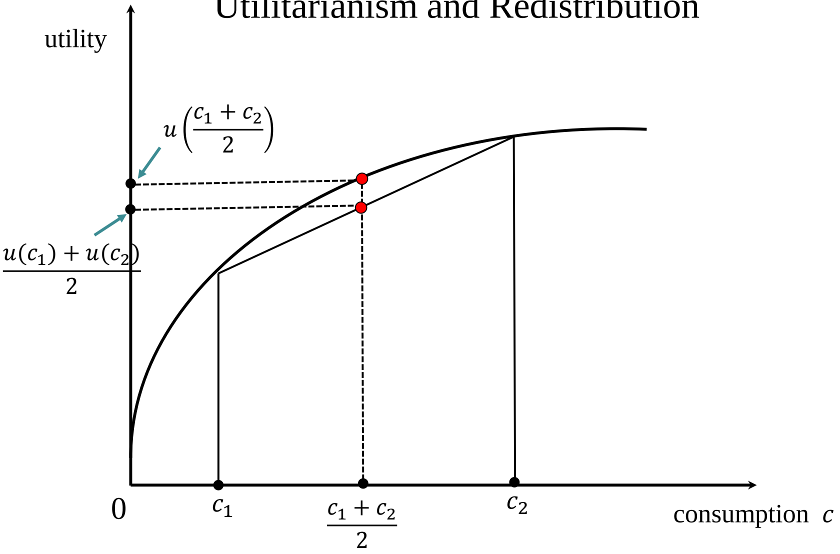
$\Rightarrow z - T(z) =$ constant for all z.

$\Rightarrow c = \bar{z} - E$ where $\bar{z} = \int zh(z)dz$ average income.

100% marginal tax rate. Perfect equalization of after-tax income.

Utilitarianism with decreasing marginal utility leads to perfect egalitarianism [Edgeworth, 1897]

Utilitarianism and Redistribution



ISSUES WITH SIMPLE MODEL

1) **No behavioral responses:** Obvious missing piece: 100% redistribution would destroy incentives to work and thus the assumption that z is exogenous is unrealistic

$\Rightarrow$ Optimal income tax theory incorporates behavioral responses (Mirrlees REStud '71): **equity-efficiency trade-off**

2) **Issue with Utilitarianism:** Even absent behavioral responses, many people would object to 100% redistribution [perceived as confiscatory]

$\Rightarrow$ Citizens' views on fairness impose **bounds** on redistribution.

The issue is the restricted nature of social preferences that can be captured by most social welfare functions.

We will discuss preferences for redistribution in another lecture!

MIRRLEES OPTIMAL INCOME TAX MODEL

We will solve the Mirrleesian model later. For now, let's look at the spirit of optimal tax evolution.

1) Standard labor supply model: Individual maximizes $u(c, l)$ subject to $c = wl - T(wl)$ where c consumption, l labor supply, w wage rate, $T(.)$ nonlinear income tax $\Rightarrow$ taxes affect labor supply

2) Individuals differ in ability w: w distributed with density $f(w)$.

3) Govt social welfare maximization: Govt maximizes

$$SWF = \int G(u(c, l))f(w)dw$$

($G(.)$ $\uparrow$ concave) subject to

(a) budget constraint $\int T(wl)f(w)dw \geq E$ (multiplier λ)

(b) individuals' labor supply l depends on $T(.)$

MIRRLEES MODEL RESULTS

Optimal income tax trades-off redistribution and efficiency (as tax based on w only not feasible)

$\Rightarrow$ $T(.) < 0$ at bottom (transfer) and $T(.) > 0$ further up (tax) [full integration of taxes/transfers]

Mirrlees formulas complex, only a couple fairly general results:

1) $0 \leq T'(.) \leq 1$, $T'(.) \geq 0$ is non-trivial (rules out EITC) [Seade '77]

2) Marginal tax rate $T'(.)$ should be zero at the top (if skill distribution bounded) [Sadka '76-Seade '77]

3) If everybody works and lowest $wl > 0$, $T'(.) = 0$ at bottom

HISTORY: BEYOND MIRRLEES

Mirrlees '71 had a huge impact on information economics: models with asymmetric information in contract theory

Discrete 2-type version of Mirrlees model developed by Stiglitz JpubE '82 with individual FOC replaced by Incentive Compatibility constraint [high type should not mimick low type]

Till late 1990s, Mirrlees results not closely connected to empirical tax studies and little impact on tax policy recommendations

Since late 1990s, Diamond AER'98, Piketty '97, Saez ReStud '01 have connected Mirrlees model to practical tax policy / empirical tax studies

[new approach summarized in Diamond-Saez JEP'11 and Piketty-Saez Handbook'13]

INTENSIVE LABOR SUPPLY CONCEPTS

$$\max_{c,z} u(\underset{+}{c}, \underset{-}{z}) \text{ subject to } c = z \cdot (1 - \tau) + R$$

Imagine a linearized budget constraint: R is virtual income (why virtual?) and τ marginal tax rate.

FOC in $c, z \Rightarrow (1-\tau)u_c + u_z = 0 \Rightarrow$ Marshallian labor supply $z = z(1-\tau, R)$

$$\text{Uncompensated elasticity} \quad \varepsilon^u = \frac{(1-\tau)}{z}\frac{\partial z}{\partial(1-\tau)}$$

$$\text{Income effects} \quad \eta = (1-\tau)\frac{\partial z}{\partial R} \leq 0$$

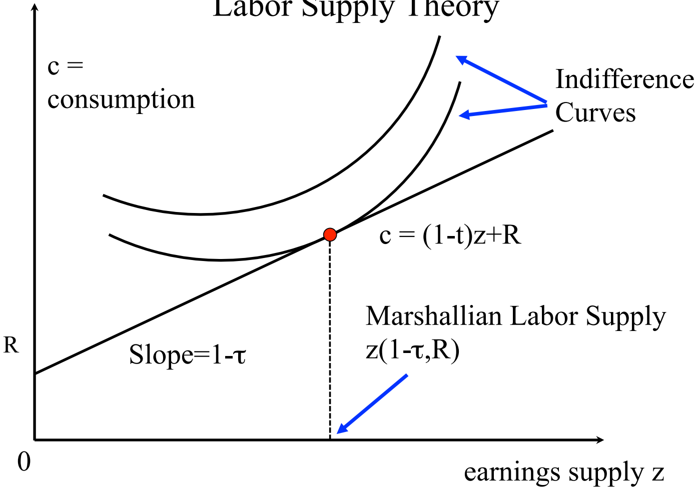
INTENSIVE LABOR SUPPLY CONCEPTS (II)

Substitution effects: Hicksian labor supply: $z^c(1-\tau, u)$ minimizes cost needed to reach u given slope $1-\tau \Rightarrow$

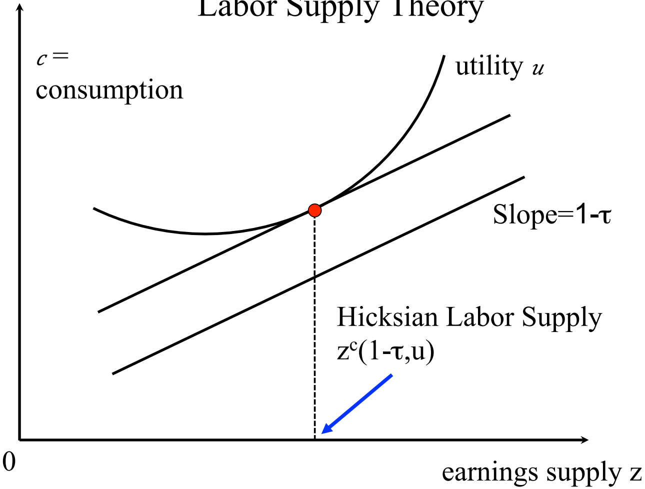
$$\text{Compensated elasticity} \quad \varepsilon^c = \frac{(1-\tau)}{z}\frac{\partial z^c}{\partial(1-\tau)} > 0$$

$$\text{Slutsky equation} \quad \frac{\partial z}{\partial(1-\tau)} = \frac{\partial z^c}{\partial(1-\tau)} + z\frac{\partial z}{\partial R} \Rightarrow \varepsilon^u = \varepsilon^c + \eta$$

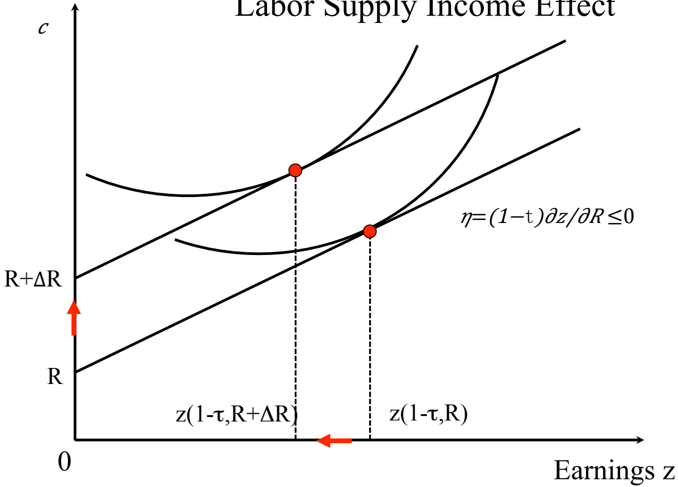
Labor Supply Theory



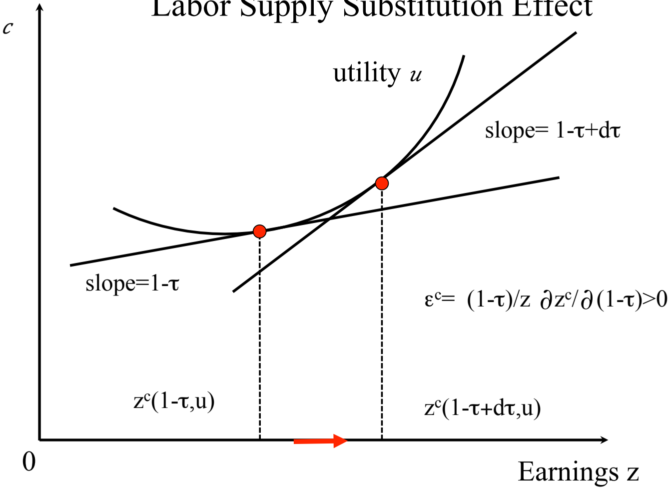
Labor Supply Theory



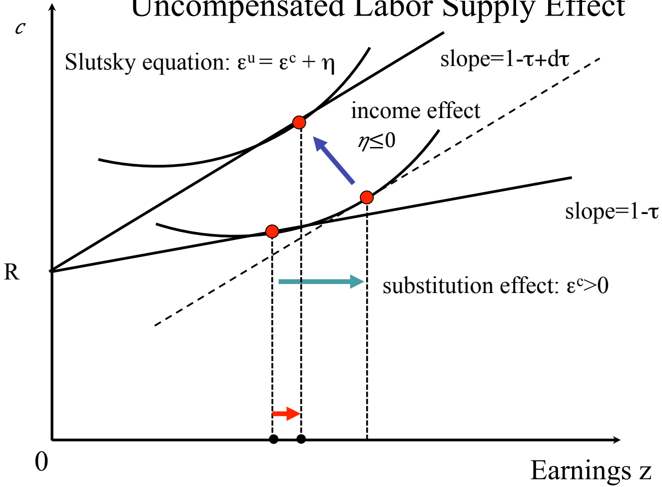
Labor Supply Income Effect



Labor Supply Substitution Effect



Uncompensated Labor Supply Effect



Labor Supply Effects of Taxes and Transfers

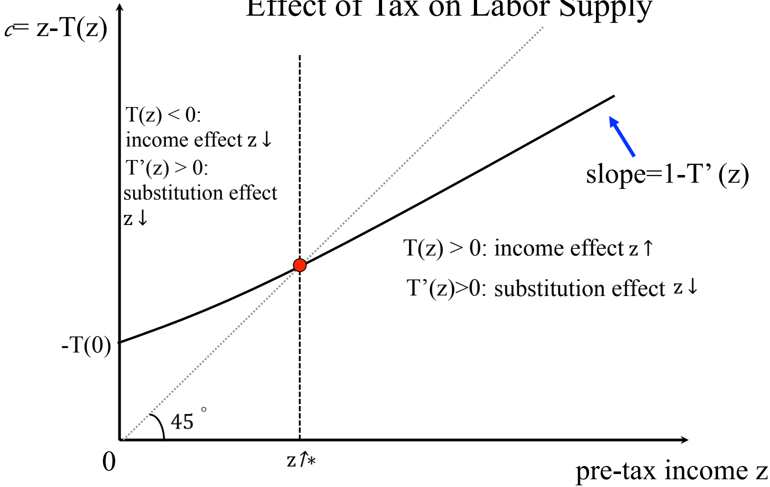
Taxes and transfers change the slope $1 - T'(z)$ of the budget constraint and net disposable income $z - T(z)$ (relative to the no tax situation where $c = z$)

Positive MTR $T'(z) > 0$ reduces labor supply through substitution effects

Net transfer ($T(z) < 0$) reduces labor supply through income effects

Net tax ($T(z) > 0$) increases labor supply through income effects

Effect of Tax on Labor Supply



WELFARE EFFECT OF SMALL TAX REFORM

Indirect utility: $V(1-\tau, R) = \max_z u((1-\tau)z + R, z)$ where R is virtual income intercept

Small tax reform: $d\tau$ and dR:

$dV = u_c \cdot [-zd\tau + dR] + dz \cdot [(1-\tau)u_c + u_z] = u_c \cdot [-zd\tau + dR]$

Envelope theorem: no effect of dz on V because z is already chosen to maximize utility ($(1-\tau)u_c + u_z = 0$)

$[-zd\tau + dR]$ is the mechanical change in disposable income due to tax reform

Welfare impact of a small tax reform is given by u_c times the money metric mechanical change in tax

WELFARE EFFECT OF SMALL TAX REFORM (II)

!! Remains true of any nonlinear tax system $T(z)$

Just need to look at $dT(z)$, mechanical change in taxes, or dT_i for agent i.

dV_i = Welfare impact is $-u_c dT(z_i)$.

When is the welfare impact not just the mechanical change in disposable income?

Envelope Theorem: For a constrained problem

$$V(\theta) = \max_x F(x, \theta) \quad \text{s.t.} \quad c \geq G(x, \theta)$$

$$V'(\theta) = \frac{\partial F}{\partial \theta}(x^*(\theta), \theta) - \lambda^*(\theta) \frac{\partial G}{\partial \theta}(x^*(\theta), \theta)$$

SOCIAL WELFARE FUNCTIONS (SWF)

Welfarism = social welfare based solely on individual utilities

Any other social objective will lead to Pareto dominated outcomes in some circumstances (Kaplow and Shavell JPE'01) Why?

Most widely used welfarist SWF:

1) Utilitarian: $SWF = \int_i u^i$

2) Rawlsian (also called Maxi-Min): $SWF = \min_i u^i$

3) $SWF = \int_i G(u^i)$ with $G(.) \uparrow$ and concave, e.g., $G(u) = u^{1-\gamma}/(1-\gamma)$
(Utilitarian is $\gamma = 0$, Rawlsian is $\gamma = \infty$)

4) General Pareto weights: $SWF = \int_i \mu_i \cdot u^i$ with $\mu_i \geq 0$ exogenously given

SOCIAL MARGINAL WELFARE WEIGHTS

Key sufficient statistics in optimal tax formulas are **Social Marginal Welfare Weights** for each individual:

Social Marginal Welfare Weight on individual i is $g_i = G'(u^i)u^i_c/\lambda$ (λ multiplier of govt budget constraint) measures \$ value for govt of giving \$1 extra to person i

No income effects $\Rightarrow \int_i g_i = 1$: giving \$1 to all costs \$1 (population has measure 1) and increase SWF (in \$ terms) by $\int_i g_i$

g_i typically depend on tax system (endogenous variable)

Utilitarian case: g_i decreases with z_i due to decreasing marginal utility of consumption

Rawlsian case: g_i concentrated on most disadvantaged (typically those with $z_i = 0$)

OPTIMAL LINEAR TAX RATE: FORMULA

Government chooses τ to maximize

$$\int_i G[u^i((1-\tau)z^i + \tau Z(1-\tau), z^i)]$$

Govt FOC (using the envelope theorem as z^i maximizes u^i):

$$0 = \int_i G'(u^i)u_c^i \cdot \left[-z^i + Z - \tau \frac{dZ}{d(1-\tau)}\right],$$

$$0 = \int_i G'(u^i)u_c^i \cdot \left[(Z - z^i) - \frac{\tau}{1-\tau} eZ\right],$$

First term $(Z - z^i)$ is mechanical redistributive effect of $d\tau$, second term is efficiency cost due to behavioral response of Z

$\Rightarrow$ we obtain the following optimal linear income tax formula

$$\tau = \frac{1-\bar{g}}{1-\bar{g}+e} \quad \text{with} \quad \bar{g} = \frac{\int g_i \cdot z_i}{Z \cdot \int g_i}, \quad g_i = G'(u^i)u_c^i$$

OPTIMAL LINEAR TAX RATE: FORMULA

$$\tau = \frac{1-\bar{g}}{1-\bar{g}+e} \quad \text{with} \quad \bar{g} = \frac{\int g_i \cdot z_i}{Z \cdot \int g_i}, \quad g_i = G'(u^i)u_c^i$$

$0 \leq \bar{g} < 1$ if g_i is decreasing with z_i (social marginal welfare weights fall with z_i).

$\bar{g}$ low when (a) inequality is high, (b) $g^i \downarrow$ sharply with z^i

Formula captures the equity-efficiency trade-off **robustly** ($\tau \downarrow \bar{g}$, $\tau \downarrow e$)

Rawlsian case: $g_i \equiv 0$ for all $z_i > 0$ so $\bar{g} = 0$ and $\tau = 1/(1+e)$

OPTIMAL TOP INCOME TAX RATE (SAEZ '01)

Consider constant MTR τ above fixed z^*. Goal is to derive optimal τ

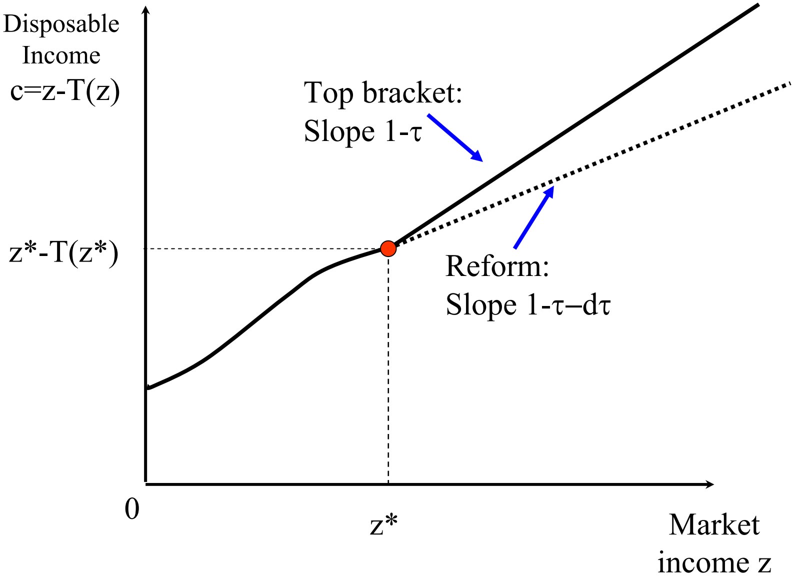
Assume w.l.o.g there is a continuum of measure one of individuals above z^*

Let $z(1-\tau)$ be their average income [depends on net-of-tax rate $1-\tau$], with elasticity $e = [(1-\tau)/z] \cdot dz/d(1-\tau)$

! Careful, what is e?

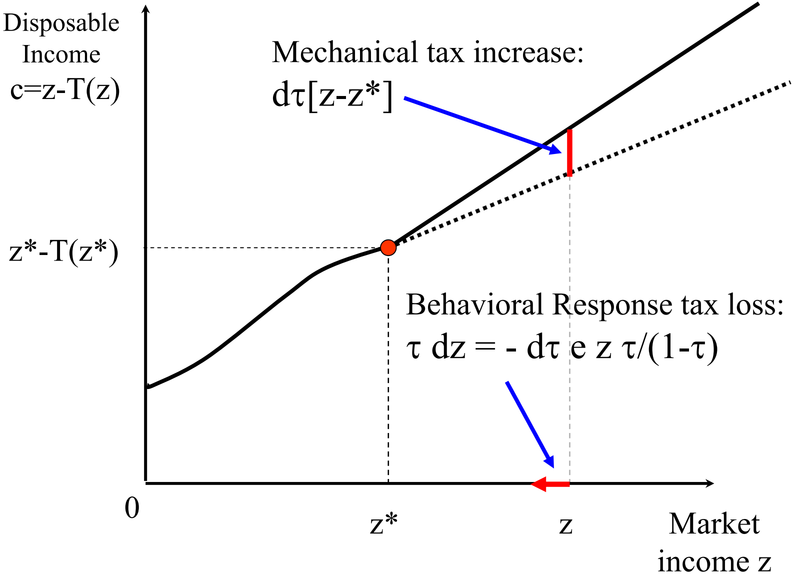
Note that e is a mix of income and substitution effects (see Saez '01)

Optimal Top Income Tax Rate (Mirrlees ’71 model)



Source: Diamond and Saez JEP'11

Optimal Top Income Tax Rate (Mirrlees '71 model)



Source: Diamond and Saez JEP'11

OPTIMAL TOP INCOME TAX RATE

Consider small $d\tau > 0$ reform above z^*.

1) **Mechanical increase** in tax revenue:

$$dM = [z - z^*]d\tau$$

2) **Welfare effect:**

$$dW = -\bar{g}dM = -\bar{g}[z - z^*]d\tau$$

where $\bar{g}$ is the social marginal welfare weight for top earners

3) **Behavioral response** reduces tax revenue:

$$dB = \tau \cdot dz = -\tau \frac{dz}{d(1-\tau)} d\tau = -\frac{\tau}{1-\tau} \cdot \frac{1-\tau}{z} \frac{dz}{d(1-\tau)} \cdot zd\tau$$

$$\Rightarrow dB = -\frac{\tau}{1-\tau} \cdot e \cdot zd\tau$$

OPTIMAL TOP INCOME TAX RATE

$$dM + dW + dB = d\tau \left[(1-\bar{g})[z - z^*] - e \frac{\tau}{1-\tau} z \right]$$

Optimal τ such that $dM + dW + dB = 0 \Rightarrow$

$$\frac{\tau}{1-\tau} = \frac{(1-\bar{g})[z-z^*]}{e \cdot z}$$

$$\tau = \frac{1-\bar{g}}{1-\bar{g}+a \cdot e} \quad \text{with} \quad a = \frac{z}{z-z^*}$$

Optimal $\tau \downarrow \bar{g}$ [redistributive tastes]

Optimal $\tau \downarrow$ with e [efficiency]

Optimal $\tau \downarrow a$ [thinness of top tail]

SUFFICIENT STATS FORMULA

Pause for a bit: did we say anything about underlying characteristics of people?

Note how general the formula is!

Sufficient statistics, observables only.

ZERO TOP RATE RESULT

Suppose top earner earns z^T

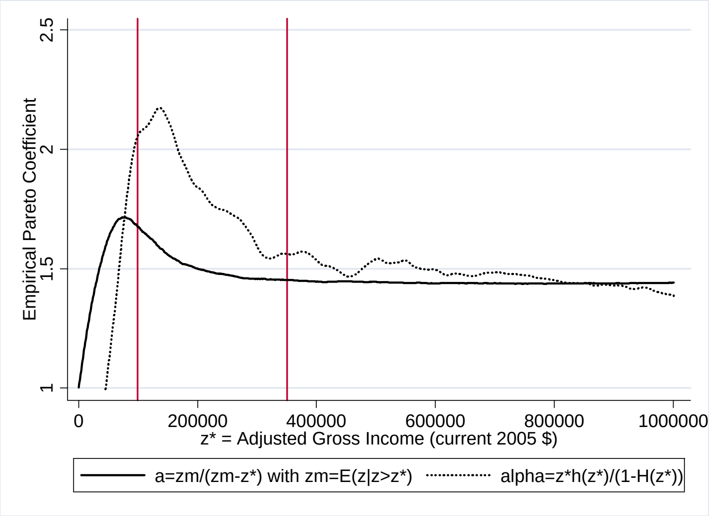
When $z^* \to z^T \Rightarrow z \to z^T$

$$dM = d\tau[z - z^*] << dB = d\tau \cdot e \cdot \frac{\tau}{1-\tau} z \quad \text{when} \quad z^* \to z^T$$

Intuition: extra tax applies only to earnings above z^* but behavioral response applies to full $z \Rightarrow$

Optimal τ should be zero when z^* close to z^T (Sadka-Seade zero top rate result) but result applies only to top earner

Top is uncertain: If actual distribution is finite draw from an underlying Pareto distribution then expected revenue maximizing rate is $1/(1 + a \cdot e)$ (Diamond and Saez JEP'11)



Source: Diamond and Saez JEP'11

OPTIMAL TOP INCOME TAX RATE

Empirically: $a = z/(z - z^*)$ very stable above $z^* = \$400K$

Pareto distribution $1 - F(z) = (k/z)^\alpha$, $f(z) = \alpha \cdot k^\alpha / z^{1+\alpha}$, with α Pareto parameter

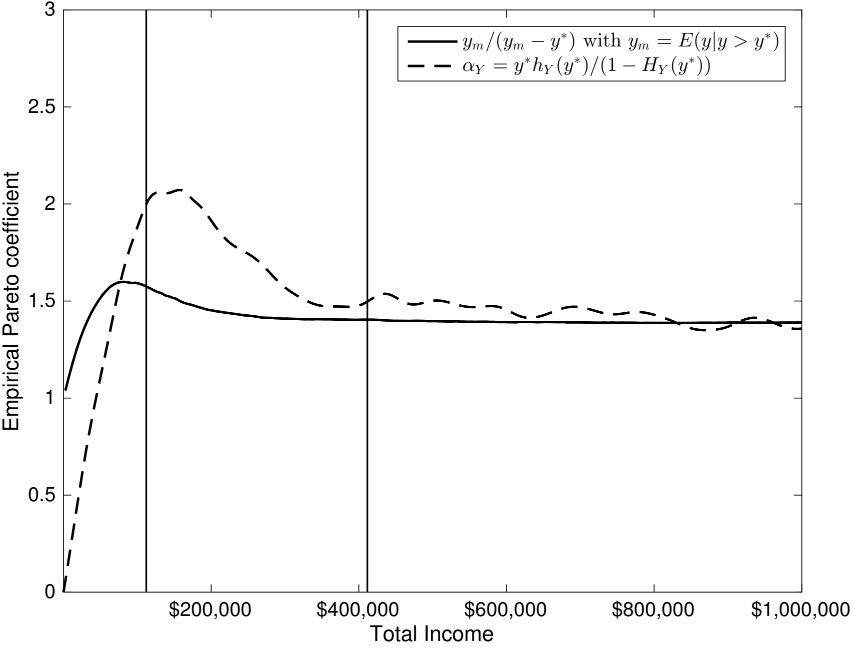
$$z(z^*) = \frac{\int_{z^*}^{\infty} s f(s) ds}{\int_{z^*}^{\infty} f(s) ds} = \frac{\int_{z^*}^{\infty} s^{-\alpha} ds}{\int_{z^*}^{\infty} s^{-\alpha-1} ds} = \frac{\alpha}{\alpha - 1} \cdot z^*$$

$\alpha = z/(z - z^*) = a$ measures *thinness* of top tail of the distribution

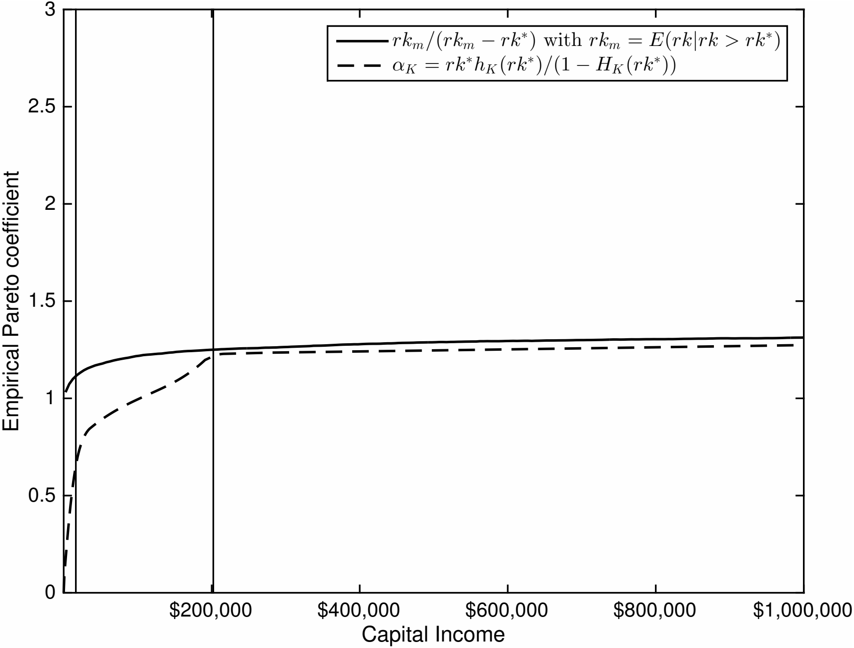
Empirically $a \in (1.5, 3)$, US has $a = 1.5$, Denmark has $a = 3$

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + a \cdot e}$$

Only difficult parameter to estimate is e



3
2.5
2
1.5
1
0.5
0
Empirical Pareto coefficient
$y_m/(y_m - y^*)$ with $y_m = E(y|y > y^*)$
$\alpha_Y = y^* h_Y(y^*)/(1 - H_Y(y^*))$
\$200,000
\$400,000
\$600,000
\$800,000
\$1,000,000
Total Income



$rk_m/(rk_m - rk^*)$ with $rk_m = E(rk|rk > rk^*)$
$\alpha_K = rk^* h_K(rk^*)/(1 - H_K(rk^*))$
Empirical Pareto coefficient
Capital Income
0
0.5
1
1.5
2
2.5
3
\$200,000
\$400,000
\$600,000
\$800,000
\$1,000,000

TAX REVENUE MAXIMIZING TAX RATE

Utilitarian criterion with $u_c \to 0$ when $c \to \infty \Rightarrow \bar{g} \to 0$ when $z^* \to \infty$

Rawlsian criterion (maximize utility of worst off person) $\Rightarrow \bar{g} = 0$ for any $z^* > \min(z)$

In the end, $\bar{g}$ reflects the value that society puts on marginal consumption of the rich

$\bar{g} = 0 \Rightarrow$ Tax Revenue Maximizing Rate $\tau = 1/(1 + a \cdot e)$ (upper bound on top tax rate)

Example: $a = 2$ and $e = 0.25 \Rightarrow \tau = 2/3 = 66.7\%$

Laffer linear rate is a special case with $z^* = 0$, $z^m/z^* = \infty = a/(a-1)$ and hence $a = 1$, $\tau = 1/(1+e)$

EXTENSIONS AND LIMITATIONS

1) Model includes only intensive earnings response. Extensive earnings responses [entrepreneurship decisions, migration decisions] $\Rightarrow$ Formulas can be modified

2) Model does not include **fiscal externalities**: part of the response to $d\tau$ comes from **income shifting** which affects other taxes $\Rightarrow$ Formulas can be modified

3) Model does not include **classical externalities**: (a) charitable contributions, (b) positive spillovers (trickle down) [top earners underpaid], (c) negative spillovers [top earners overpaid]

Classical general equilibrium effects on prices are NOT externalities and do not affect formulas [Diamond-Mirrlees AER '71, Saez JpubE '04]

GENERAL NON-LINEAR INCOME TAX $T(z)$

(1) Lumpsum grant given to everybody equal to $-T(0)$

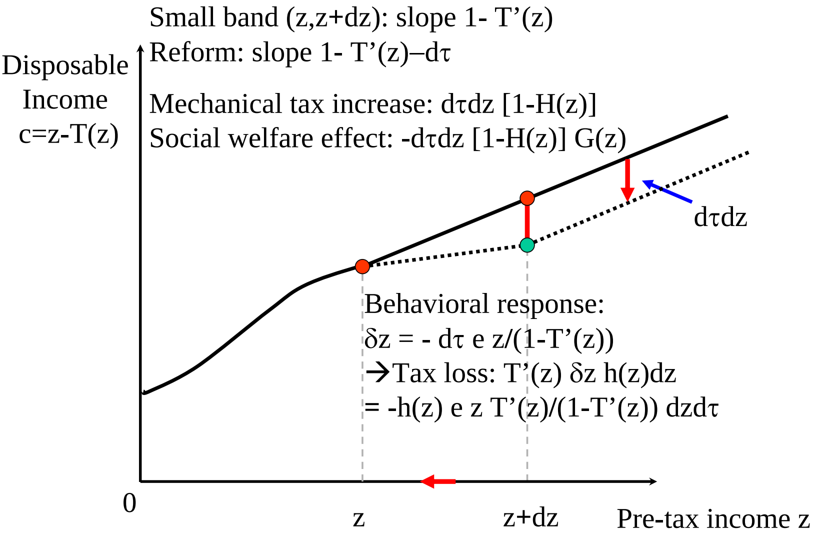
(2) Marginal tax rate schedule $T'(z)$ describing how (a) lump-sum grant is taxed away, (b) how tax liability increases with income

Let $H(z)$ be the income CDF [population normalized to 1] and $h(z)$ its density [endogenous to $T(.)$]

Let $g(z)$ be the social marginal value of consumption for taxpayers with income z in terms of public funds [formally $g(z) = G'(u) \cdot u_c/\lambda$]: no income effects $\Rightarrow \int g(z)h(z)dz = 1$

Redistribution valued $\Rightarrow g(z)$ decreases with z

Let $G(z)$ the *average* social marginal value of c for taxpayers with income above z [$G(z) = \int_z^\infty g(s)h(s)ds/(1-H(z))$]



Source: Diamond and Saez JEP'11

GENERAL NON-LINEAR INCOME TAX

Assume away income effects $\varepsilon^c = \varepsilon^u = e$ [Diamond AER'98 shows this is the key theoretical simplification]

Consider small reform: increase T' by $d\tau$ in small band z and $z + dz$

Mechanical effect $dM = dzd\tau[1 - H(z)]$

Welfare effect $dW = -dzd\tau[1 - H(z)]G(z)$

Behavioral effect: substitution effect δz inside small band $[z, z + dz]$:
$dB = h(z)dz \cdot T' \cdot \delta z = -h(z)dz \cdot T' \cdot d\tau \cdot z \cdot e_{(z)}/(1 - T')$

Optimum $dM + dW + dB = 0$

GENERAL NON-LINEAR INCOME TAX

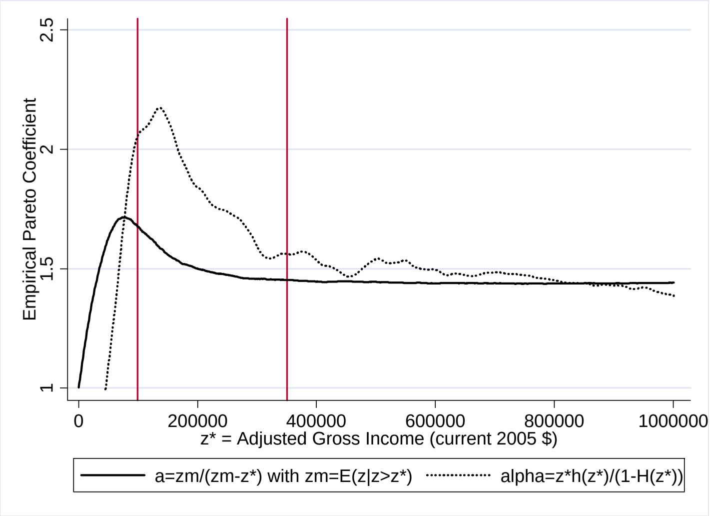
$$T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e_{(z)}}$$

1) $T'(z)$ decreases with $e_{(z)}$ (elasticity efficiency effects)

2) $T'(z)$ decreases with $\alpha(z) = (zh(z))/(1 - H(z))$ (local Pareto parameter)

3) $T'(z)$ decreases with $G(z)$ (redistributive tastes)

Asymptotics: $G(z) \to \bar{g}$, $\alpha(z) \to a$, $e_{(z)} \to e \Rightarrow$ Recover top rate formula $\tau = (1 - \bar{g})/(1 - \bar{g} + a \cdot e)$



Source: Diamond and Saez JEP'11

Negative Marginal Tax Rates Never Optimal

Suppose $T' < 0$ in band $[z, z + dz]$

Increase T' by $d\tau > 0$ in band $[z, z + dz]$: $dM + dW > 0$ and $dB > 0$ because $T'(z) < 0$

$\Rightarrow$ Desirable reform

$\Rightarrow$ $T'(z) < 0$ cannot be optimal

EITC schemes are not desirable in Mirrlees '71 model

MIRRLEES MODEL

The difference to before: we need to specify the *structural primitives.*

Key simplification is the lack of income effects (Diamond, 1998). We look into income effects next time.

Individual utility: $c - v(l)$, l is labor supply.

Skill n is exogenously given, equal to marginal productivity. Earnings are $z = nl$.

Density is $f(n)$ and CDF $F(n)$ on $[0, \infty)$.

Entry into contract theory/mechanism design here: The government does not observe **skill**. Tax is based on income z, $T(z)$.

What happens if we had a tax $T(n)$ available?

Why did we not talk about this in the earlier derivations? Did we ignore the incentive compatibility constraints?

Elasticity of labor to taxes

Recall we derive elasticities on the linearized budget set. If marginal tax rate is τ, labor supply is: $l = l(n(1-\tau))$. Why the $n(1-\tau)$? Why *only* $n(1-\tau)$?

FOC of the agent for labor supply:

$$n(1-\tau) = v'(l)$$

Totally differentiate this (key thing: skill is fixed!)

$$d(n(1-\tau)) = v''(l)dl$$

$$\Rightarrow e = \frac{dl}{d(n(1-tau))}\frac{(1-\tau)n}{l} = \frac{(1-\tau)n}{lv''(l)} = \frac{v'(l)}{lv''(l)}$$

Is this compensated? uncompensated?

Direct Revelation Mechanism and Incentive Compatibility

We want to max social welfare and have exogenous revenue requirement (non transfer-related E).

We imagine a direct revelation mechanism. Every agent comes to government, reports a type n'. We assign allocations as a function of the report. $c(n')$, $z(n')$, $u(n')$. Why are we not assigning labor $l(n')$?

What are the constraints in this problem?

Feasibility (net resources sum to zero): $\int_n c_n f(n) dn \geq n l_n f(n) dn - E$.

Incentive compatibility:

Direct Revelation Mechanism and Incentive Compatibility

We want to max social welfare and have exogenous revenue requirement (non transfer-related E).

We imagine a direct revelation mechanism. Every agent comes to government, reports a type n'. We assign allocations as a function of the report. $c(n')$, $z(n')$, $u(n')$. Why are we not assigning labor $l(n')$?

What are the constraints in this problem?

Feasibility (net resources sum to zero): $\int_n c_n f(n) dn \geq n l_n f(n) dn - E$.

Incentive compatibility:

$$c(n) - v\left(\frac{z(n)}{n}\right) \geq c(n') - v\left(\frac{z(n')}{n}\right) \; \forall n, n'$$

That's a lot of constraints!

Envelope Theorem and First order Approach

Replace the infinity of constraints with agents' first-order condition. If we take derivative of utility wrt type n at *truth-telling*

$$\frac{du_n}{dn} = \left(c'(n) - \frac{z'(n)}{n} v'\left(\frac{z(n)}{n}\right)\right) \frac{dn'}{dn} + \frac{z(n)}{n^2} v'\left(\frac{z(n)}{n}\right)$$

What if report is optimally chosen?

Envelope condition:

$$\frac{du_n}{dn} = \frac{l_n v'(l_n)}{n}$$

Will replace infinity of constraints.

Is necessary, but what about sufficiency?

Full Optimization Program

$\max_{c_n, u_n, z_n} \int_n G(u_n) f(n) dn \quad \text{s.t.} \quad \int_n c_n f(n) dn \leq \int_n n l_n f(n) dn - E$

and s.t. $\frac{du_n}{dn} = \frac{l_n v'(l_n)}{n}$

State variable: u_n.

Control variables: l_n, with $c_n = u_n + v(l_n)$.

Why am I suddenly saying l_n is a control?

Use optimal control.

Hamiltonian and Optimal Control

The Hamiltonian is:

$$H = [G(u_n) + p \cdot (nl_n - u_n - v(l_n))]f(n) + \phi(n) \cdot \frac{l_n v'(l_n)}{n}$$

p: multiplier on the resource constraint.

$\phi(n)$: multiplier on the envelope condition ("costate"). Depends on n!

FOCs:

$$\frac{\partial H}{\partial l_n} = p \cdot [n - v'(l_n)]f(n) + \frac{\phi(n)}{n} \cdot [v'(l_n) + l_n v''(l_n)] = 0$$

$$\frac{\partial H}{\partial u_n} = [G'(u_n) - p]f(n) = -\frac{d\phi(n)}{dn}$$

Transversality: $\lim_{n\to\infty} \phi(n) = 0$ and $\phi(0) = 0$.

Rearranging the FOCs

Take the integral of the FOC wrt u_n to solve for $\phi(n)$:

$$-\phi(n) = \int_n^\infty [p - G'(u_m)]f(m)dm$$

Integrate this same FOC over the full space, using transversality conditions:

$$p = \int_0^\infty G'(u_n)f(m)dm$$

What does this say?

How can we make the tax rate appear? Use the agent's FOC.

$$n - v'(l_n) = nT'(z_n)$$

Obtaining the Optimal Tax Formula

Recall that $e = \frac{(1-T'(z_n))n}{lv''(l)}$

Rearranging the last term in the FOC for l_n:

$$[v'(l_n) + l_n v''(l_n)]/n = [1 - T'(z_n)][1 + 1/e]$$

Let $g_m \equiv G'(u_m)/p$ be the marginal social welfare weight on type m.

Then, the FOC for l_n becomes:

$$\frac{T'(z_n)}{1 - T'(z_n)} = \left(1 + \frac{1}{e}\right) \cdot \left(\frac{\int_n^\infty (1 - g_m) dF(m)}{nf(n)}\right)$$

This is the Diamond (1998) formula.

What is different from the previously derived formula *à la* Saez (2001)?

Let's go from types to observable income

How do we go from type distribution to income distribution?

Under linearized tax schedule, earnings are a function $z_n = nl(n(1-\tau))$.

How do earnings vary with type?

$$\frac{dz_n}{dn} = l + (1-\tau)n\frac{dl}{d(m(1-\tau))} = l_n \cdot (1+e)$$

(intuition?)

Let $h(z)$ be the density of earnings, with CDF $H(z)$. The following relation must hold:

$$h(z_n)dz_n = f(n)dn$$

$$f(n) = h(z_n)l_n(1+e) \Rightarrow ng(n) = z_n h(z_n)(1+e)$$

Let's substitute income distributions for type distributions in the formula.

Optimal Tax Formula with No Income Effects

$$\frac{T'(z_n)}{1-T'(z_n)} = \frac{1}{e}\left(\frac{\int_n^\infty (1-g_m)dF(m)}{z_n h(z_n)}\right) = \frac{1}{e}\left(\frac{1-H(z_n)}{z_n h(z_n)}\right)\cdot(1-G(z_n))$$

where:

$$G(z_n) = \frac{\int_n^\infty g_m dF(m)}{1-F(n)}$$

is the average marginal social welfare weight on individuals with income above z_n.

Change of variables to income distributions:

$$G(z_n) = \frac{\int_{z_n}^\infty g_m dH(z_m)}{1-H(z_n)}$$

NUMERICAL SIMULATIONS

$H(z)$ [and also $G(z)$] endogenous to $T(.)$. Calibration method (Saez Restud '01):

Specify utility function (e.g. constant elasticity):

$$u(c,z) = c - \frac{1}{1+\frac{1}{e}} \cdot \left(\frac{z}{n}\right)^{1+\frac{1}{e}}$$

Individual FOC $\Rightarrow z = n^{1+e}(1-T')^e$

Calibrate the exogenous skill distribution $F(n)$ so that, using **actual** $T'(.)$, you recover **empirical** $H(z)$

Use Mirrlees '71 tax formula (expressed in terms of $F(n)$) to obtain the optimal tax rate schedule T'.

NUMERICAL SIMULATIONS

$$\frac{T'(z(n))}{1-T'(z(n))} = \left(1+\frac{1}{e}\right)\left(\frac{1}{nf(n)}\right)\int_n^\infty \left[1-\frac{G'(u(m))}{\lambda}\right] f(m)dm,$$

Iterative Fixed Point method: start with T'_0, compute $z^0(n)$ using individual FOC, get $T^0(0)$ using govt budget, compute $u^0(n)$, get λ using $\lambda = \int G'(u)f$, use formula to estimate T'_1, iterate till convergence

Fast and effective method (Brewer-Saez-Shepard '10)

NUMERICAL SIMULATION RESULTS

$$T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e_{(z)}}$$

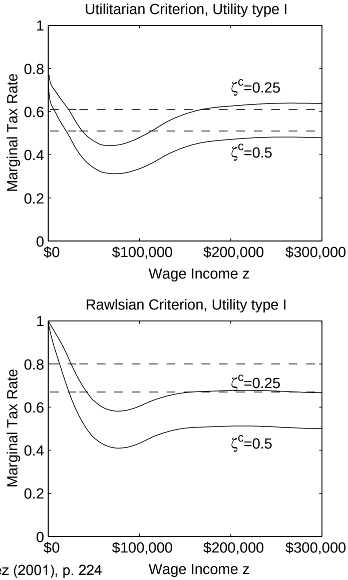
Take utility function with e constant

2) $\alpha(z) = (zh(z))/(1 - H(z))$ is inversely U-shaped empirically

3) $1 - G(z)$ increases with z from 0 to 1 ($\bar{g} = 0$)

$\Rightarrow$ Numerical optimal $T'(z)$ is U-shaped with z: reverse of the general results $T' = 0$ at top and bottom [Diamond AER'98 gives theoretical conditions to get U-shape]

FIGURE 5 − Optimal Tax Simulations



Source: Saez (2001), p. 224

REFERENCES (for lectures 2 and 3)

Akerlof, G. "The Economics of Tagging as Applied to the Optimal Income Tax, Welfare Programs, and Manpower Planning", American Economic Review, Vol. 68, 1978, 8-19. (web)

Atkinson, A.B. and J. Stiglitz "The design of tax structure: Direct versus indirect taxation", Journal of Public Economics, Vol. 6, 1976, 55-75. (web)

Besley, T. and S. Coate "Workfare versus Welfare: Incentives Arguments for Work Requirements in Poverty-Alleviation Programs", American Economic Review, Vol. 82, 1992, 249-261. (web)

Boskin, M. and E. Sheshinski "Optimal tax treatment of the family: Married couples", Journal of Public Economics, Vol. 20, 1983, 281-297 (web)

Brewer, M., E. Saez, and A. Shephard "Means Testing and Tax Rates on Earnings", in The Mirrlees Review: Reforming the Tax System for the 21st Century, Oxford University Press, 2010. (web)

Cremer, H., F. Gahvari, and N. Ladoux "Externalities and optimal taxation", Journal of Public Economics, Vol. 70, 1998, 343-364. (web)

Deaton, A. "Optimal Taxes and the Structure of Preferences", Econometrica, Vol. 49, 1981, 1245-1260 (web)

Diamond, P. "A many-person Ramsey tax rule", Journal of Public Economics, Vol.4, 1975, 335-342. (web)

Diamond, P. "Income Taxation with Fixed Hours of Work"Journal of Public Economics, Vol. 13, 1980, 101-110. (web)

Diamond, P. "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates", American Economic Review, Vol. 88, 1998, 83-95. (web) Skim this for historical reasons.

Diamond, P. and E. Saez "From Basic Research to Policy Recommendations: The Case for a Progressive Tax", Journal of Economic Perspectives, 25(4), 2011, 165-190. (web)

Edgeworth, F. "The Pure Theory of Taxation", The Economic Journal, Vol. 7, 1897, 550-571. (web)

Kaplow, L. "On the undesirability of commodity taxation even when income taxation is not optimal", Journal of Public Economics, Vol. 90, 2006, 1235-1250. (web)

Kaplow, L. *The Theory of Taxation and Public Economics.* Princeton University Press, 2008.

Kaplow, L. and S. Shavell "Any Non-welfarist Method of Policy Assessment Violates the Pareto Principle," Journal of Political Economy, 109(2), (April 2001), 281-286 (web)

Kleven, H., C. Kreiner and E. Saez "The Optimal Income Taxation of Couples", Econometrica, Vol. 77, 2009, 537-560. (web)

Laroque, G. "Indirect Taxation is Superfluous under Separability and Taste Homogeneity: A Simple Proof", Economic Letters, Vol. 87, 2005, 141-144. (web)

Lehmann, E., L. Simula, A. Trannoy "Tax Me if You Can! Optimal Nonlinear Income Tax between Competing Governments," Quarterly Journal of Economics 129(4), 2014, 1995-2030. (web)

Mankiw, G. and M. Weinzierl “The Optimal Taxation of Height: A Case Study of Utilitarian Income Redistribution”, AEJ: Economic Policy, Vol. 2, 2010, 155-176. (web)

Mirrlees, J. “An Exploration in the Theory of Optimal Income Taxation”, Review of Economic Studies, Vol. 38, 1971, 175-208. (web) Read it for historical reasons, it is not very easy to follow.

Mirrlees, J. “Migration and Optimal Income Taxes”, Journal of Public Economics, Vol. 18, 1982, 319-341. (web)

Nichols, A. and R. Zeckhauser“Targeting Transfers Through Restrictions on Recipients”, American Economic Review, Vol. 72, 1982, 372-377. (web)

Piketty, Thomas and Emmanuel Saez “Optimal Labor Income Taxation,” Handbook of Public Economics, Volume 5, Amsterdam: Elsevier-North Holland, 2013. (web)

Piketty, Thomas, Emmanuel Saez, and Stefanie Stantcheva "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", American Economic Journal: Economic Policy, 6(1), 2014, 230-271 (web)

Sadka, E. “On Income Distribution, Incentives Effects and Optimal Income Taxation”, Review of Economic Studies, Vol. 43, 1976, 261-268. (web)

Saez, E. “Using Elasticities to Derive Optimal Income Tax Rates”, Review of Economics Studies, Vol. 68, 2001, 205-229. (web)

Saez, E. “The Desirability of Commodity Taxation under Non-linear Income Taxation and Heterogeneous Tastes”, Journal of Public Economics, Vol. 83, 2002, 217-230. (web)

Sandmo, A. “Optimal Taxation in the Presence of Externalities”, The Swedish Journal of Economics, Vol. 77, 1975, 86-98. (web)

Seade, Jesus K. “On the shape of optimal tax schedules.” Journal of public Economics 7.2 (1977): 203-235. (web)

Stiglitz, J. “Self-selection and Pareto Efficient Taxation”, Journal of Public Economics, Vol. 17, 1982, 213-240. (web)