Economics 2450A: Public Economics
Section 8: Optimal Minimum Wage and Introduction to Capital Taxation

Matteo Paradisi

November 1, 2016

In this Section we develop a theoretical analysis of optimal minimum wage policy in a perfectly competitive labor market following Lee and Saez (2012).

1 Optimal Minimum Wage

We study a model with extensive and intensive margin labor supply responses where wages are endogenous. Suppose output is produced through a constant return to scale production function $F(h_1, h_2)$ where h_1 and h_2 are low and high-skilled workers respectively. Profits are given by $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$ and wages are equal to the marginal product of labor:

$$w_i = \frac{\partial F(h_1, h_2)}{\partial h_i} \tag{1}$$

A mass 1 of individuals has three labor supply options: i) not work and earn zero income, ii) work in occupation 1 and get w_1, iii) work in occupation 2 and earn w_2. Individuals are heterogeneous in their tastes for work. Every individual faces a vector $\theta = (\theta_1, \theta_2)$ of work costs that is smoothly distributed across the entire population according to $H(\theta)$ with support Θ. The government perfectly observes the wage w_i, but does not observe the cost of working. There are no savings and after tax income equals consumption such that $c_i = w_i - T_i$. Suppose there are no income effects and utility is linear in consumption:

$$u_i = c_i - \theta_i$$

The subset of individuals choosing occupation i is $\Theta_i = \{\theta \in \Theta | u_i = \max_j u_j\}$. The fraction of the population working in occupation i is $h_i(c) = |\Theta_i|$ and is a function of $c = (c_0, c_1, c_2)$. The tax system defines a competitive equilibrium (h_1, h_2, w_1, w_2).

Equation (1) implies that $w_2/w_1 = F_2(1, h_2/h_1)/F_1(1, h_2/h_1)$. Constant returns to scale along with decreasing marginal productivity along each skill implies that the right- hand-side is a decreasing function of h_2/h_1. Therefore, the function is invertible and the ratio h_2/h_1 can be written as a function of the wage ratio w_2/w_1: $h_2/h_1 = \rho(w_2/w_1)$ with $\rho(\cdot)$ a decreasing function. Constant returns to scale also imply that there are no profits in equilibrium. Hence $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2 = 0$ so that $w_1 + w_2 \rho(w_2/w_1) = F(1, \rho w_2/w_1)$, which defines a decreasing mapping between w_1 and w_2 so that we can express w_2 as a decreasing function of w_1: $w_2(w_1)$.

Labor supply and demand for the low-skilled labor market are $D_1(w_1)$ and $S_1(w_1)$ with $D_1'(w_1) \leq 0$ and $S_1'(w_1) \geq 0$ and are defined assuming that the market clears in the high-skilled labor market. The low-skilled labor demand elasticity is:

$$\eta_1 = -\frac{w_1}{h_1} D_1(w_1)$$

The resource constraint of the economy is:

$$h_0 c_0 + h_1 c_1 + h_2 c_2 \leq h_1 w_1 + h_2 w_2 \tag{2}$$

The government weights individual utilities through a social welfare function $G(\cdot)$ and we can write the social welfare of the economy as:

$$SW = (1 - h_1 - h_2) G(c_0) + \int_{\Theta_1} G(c_1 - \theta_1) dH(\theta) + \int_{\Theta_2} G(c_2 - \theta_2) dH(\theta) \tag{3}$$

We define social marginal welfare weights as usual $g_0 = G'(c_0)/\lambda$ and $g_i = \int_{\Theta_i} G'(c_i - \theta_i) dH(\theta) / (\lambda h_i)$, where λ is the marginal value of public funds. The concavity of the SWF implies $g_0 > g_1$ and $g_1 > g_2$. Since there are no income effects the value of transferring \$1 to everyone in the economy is \$1 and we have $\lambda = g_0 h_0 + g_1 h_1 + g_2 h_2 = 1$.

Minimum Wage with No Taxes Suppose there are no taxes and transfers, we have $c_0 = 0$, $c_1 = w_1$ and $c_2 = w_2$. Suppose the economy is at the equilibrium and the government introduces a small minimum wage above the equilibrium wage of the low-skilled market such that $\bar{w} = w_1^* + d\bar{w}$. The change will generate a drop in employment h_1. The workers who drop out of the low-skilled sector will move either to unemployment or to the high-skilled sector depending on their preferences. We will assume *efficient rationing*: the workers who involuntarily lose their low-skilled jobs due to the minimum wage are those with the least surplus from working in the low-skilled sector.[1] This is clearly the most favorable case to minimum wage policy. We establish the first result of the paper:

Proposition 1: *With no taxes/transfers, if (i) efficient rationing holds; (ii) the government values redistribution from high-skilled workers toward low-skilled workers ($g_1 > g_2$); (iii) the demand elasticity η_1 for low-skilled workers is finite; and (iv) the supply elasticity of lows-killed workers is positive, then introducing a minimum wage increases social welfare.*

Consider the changes dw_1, dw_2, dh_1 and dh_2 following the increase in the minimum wage, we have $d\Pi = \sum_i [(\partial F/\partial h_i) dh_i - w_i dh_i - h_i dw_i] = -h_1 dw_1 - h_2 dw_2$ and the no profit condition implies:

$$h_1 dw_1 + h_2 dw_2 = 0 \tag{4}$$

Therefore, the earnings gain for low-skilled people $h_1 dw_1 > 0$ is compensated by a loss in the earnings of high-skilled workers $h_2 dw_2 < 0$. The government values the transfer of resources $[g_1 - g_2] h_1 dw_1$. Under efficient rationing, positive supply elasticity and finite demand elasticity, the welfare loss due to low-skilled individuals moving to unemployment is second-order (see Figure 1).

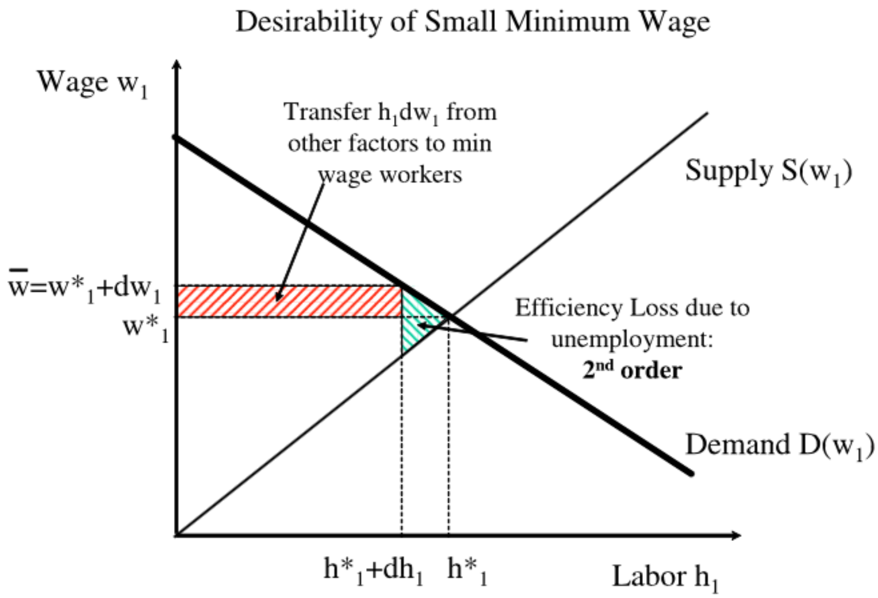
More formally, the first order condition wrt $d\bar{w}$ is:

$$\begin{aligned} \frac{dSW}{d\bar{w}} &= \left[-\frac{dh_1}{d\bar{w}} - \frac{dh_2}{d\bar{w}}\right] G(0) + \frac{dh_1}{d\bar{w}} G(0) + \frac{dh_2}{d\bar{w}} G(0) + \\ &\quad + \int_{\Theta_1} G'(c_1 - \theta_1) dH(\theta) + \frac{dw_2}{d\bar{w}} \int_{\Theta_2} G'(c_2 - \theta_2) dH(\theta) \end{aligned}$$

The second and third terms come from the assumption of perfect rationing: the workers moving to unemployment from the two occupations are those with zero surplus from working therefore the welfare loss associated to the change of occupation is zero. Also, those who drop out of occupation 1 and move to 2 are indifferent between the two and we can ignore the welfare effect associated to the change by envelope theorem. Using (4) we have $dw_2/d\bar{w} = -h_1/h_2$ and the FOC becomes:

[1] The assumption can be relaxed and a working paper version of the paper shows how the model can be derived under the assumption of uniform rationing.

Figure 1:



$$\frac{dSW}{d\bar{w}} = h_1\lambda\left[g_1 - g_2\right] > 0$$

which proves Proposition 1.

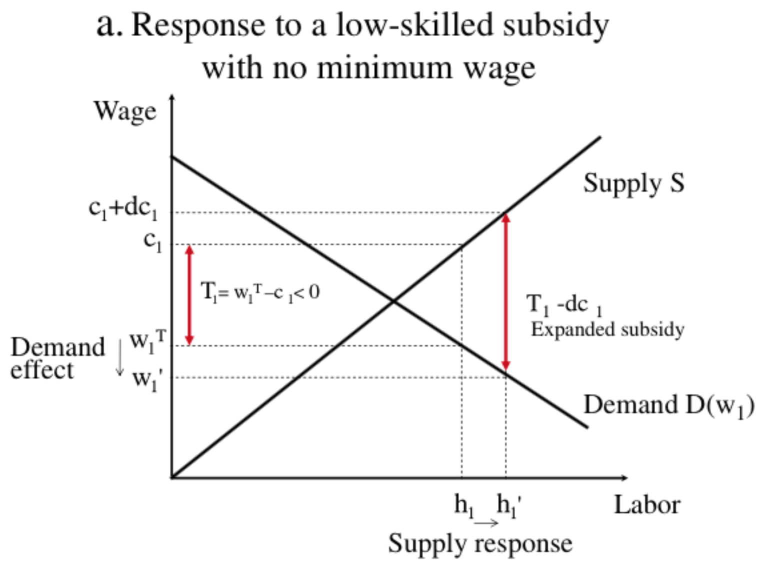
Minimum Wage with Taxes and Transfers We now assume that the government can use taxes and transfers jointly with the minimum wage policy.

Proposition 2: *Under efficient rationing, assuming $\eta_1 < \infty$, if $g_1 > 1$ at the optimal tax allocation (with no minimum wage), then introducing a minimum wage is desirable. Furthermore, at the joint minimum wage and tax optimum, we have: (i) $g_1 = 1$ (Full redistribution to low-skilled workers); (ii) $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$ (Social welfare weights average to one).*

Suppose there was no minimum wage, an attempt to increase c_1 by dc_1 while keeping c_0 and c_2 constant through an increased work subsidy provides incentives for some of the non-workers to start working in occupation 1 (extensive labor supply response) and for some of workers in occupation 2 to switch to occupation 1 (intensive labor supply response). This leads to a reduction in w_1 through demand side effects (as long as $\eta_1 < \infty$). See Figure 2.

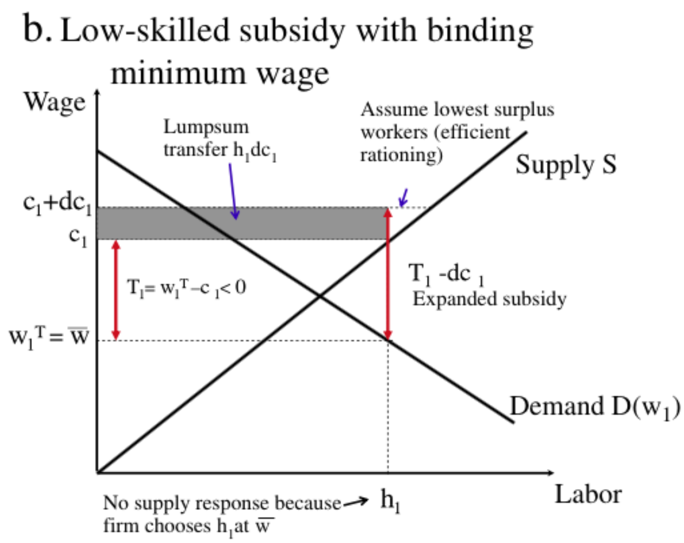
Consider the same increase in c_1 when the minimum wage was initially set at $\bar{w} = w_1^T$, where $\left(w_i^T, c_i^T\right)$ is the the optimal tax and transfer system which maximizes social welfare absent the minimum wage. Since w_1 cannot fall, labor supply responses are effectively blocked (Figure 3). Efficient rationing guarantees that individuals willing to leave occupation 1 are precisely those with the lowest surplus from working in occupation 1 relative to their next best option. Therefore, the dc_1 change is like a lump-sum tax reform and its net welfare effect is simply $\left[g_1 - 1\right] h_1 dc_1$. If $g_1 > 1$, the introduction of the minimum wage improves upon the tax/transfer optimum allocation. This result shows that under the minimum wage policy, redistribution to low-skilled workers can be made lump-sum. Furthermore, raising the lump-sum transfer to occupation 1 improves welfare as long as $g_1 > 1$ and therefore the government will find optimal

Figure 2:



to do it until $g_1 = 1$. With no behavioral responses an increase of \$1 has a welfare effect of $h_0 g_0 + h_1 g_1 + h_2 g_2$ and at the optimal the two are equal.

Figure 3:



To prove it formally, rewrite consumption in occupation i as $\Delta c_i = c_i - c_0$ and the resource constraint as $h_1 \cdot (w_1 - \Delta c_1) + h_2 \cdot (w_2 - \Delta c_2) \geq c_0$. The Lagrangian of the problem is:

$$L = SW + \lambda \left[h_1 \cdot (w_1 - \Delta c_1) + h_2 \cdot (w_2 - \Delta c_2) - c_0 \right]$$

Suppose there is a minimum wage and the government introduces a change dc_1, the wage of occupation 1 does not change because of the minimum wage and so does w_2 given that $w_2(w_1)$ (as we showed above).

As a consequence, there is no change in $h_1/h_2 = \rho(w_2/w_1)$ and no change in the levels of h_1 and h_2 since they cannot increase simultaneously. Therefore:

$$\frac{dL}{dc_1} = \int_{\Theta_1} G'(c_0 + \Delta c_1 - \theta_1)\, dH(\theta) - \lambda h_1 = \lambda [g_1 - 1] h_1$$

At the optimum it must be $g_1 = 1$. Taking the FOC wrt c_0 we have:

$$\begin{aligned} \frac{dL}{dc_0} &= (1 - h_1 - h_2) G'(c_0) + \int_{\Theta_1} G'(c_0 + \Delta c_1 - \theta_1)\, dH(\theta) + \int_{\Theta_2} G'(c_0 + \Delta c_2 - \theta_2)\, dH(\theta) - \lambda \\ &= \lambda [h_0 g_0 + h_1 g_1 + h_2 g_2 - 1] \end{aligned}$$

which proves Proposition 2.

Pareto Improving Reform In this section we review the last result in Lee and Saez (2012) that shows how minimum wage and low-skilled labor subsidies can be complementary. Suppose there are extensive margin responses only, the participation tax rate of low-skilled workers τ_1 is $1 - \tau_1 = (c_1 - c_0)/w_1$, such that $c_1 = c_0 + (1 - \tau_1) w_1$.

Proposition 3: *In a model with extensive labor supply responses only, a binding minimum wage associated with a positive tax rate on minimum wage earnings ($\tau_1 > 0$) is second-best Pareto inefficient. This result remains a-fortiori true when rationing is not efficient.*

Suppose the government reduces the minimum wage by $d\bar{w} < 0$ while keeping c_0, c_1 and c_2 constant. The change incentivizes unemployed individuals to enter occupation 1 generating a change $dh_1 > 0$ and increasing revenues since $\tau_1 > 0$. The change $dh_1 > 0$ induces a change $dw_2 > 0$. However, since $h_1 d\bar{w} + h_2 dw_2 = 0$ the mechanical effect of changes in wages is zero. Since c_0, c_1 and c_2 are constant the total effect of the government policy is only given by the increase in revenues, which is positive. Proposition 3 implies that, when labor supply responses are concentrated along the extensive margin, a minimum wage should always be associated with low-skilled work subsidies such as the EITC.

To prove it formally notice that since consumption does not change at any occupation, the utility of those who do not switch jobs is not affected. From the demand side, we have $w_2(w_1)$ with $dw_2/dw_1 = -h_1/h_2 < 0$ and hence $dw_2 > 0$. This implies that relative demand for high-skilled work $h_2/h_1 = \rho(w_2/w_1)$ decreases as $\rho(\cdot)$ is decreasing. Because $c_2 - c_0$ remains constant, and labor supply is only along the extensive margin, the supply of high-skilled workers is unchanged so that $dh_2 = 0$, which then implies that $dh_1 > 0$. The dh_1 individuals shifting from no work to low-skilled work are weakly better-off because they were by definition rationed by the minimum wage (strictly better off in case of inefficient rationing). The government budget is $h_1(w_1 - \Delta c_1) + h_2(w_2 - \Delta c_2) - c_0 \geq 0$. Therefore, the net effect of the reform on the budget is: $dh_1 \cdot (w_1 - \Delta c_1) + h_1 dw_1 + h_2 dw_2 = dh_1 \tau_1 w_1 > 0$. Thus, with $\tau_1 > 0$, the reform creates a budget surplus which can be used to increase c_0 and improve everybody's welfare (with no behavioral response effects), a Pareto improvement.

References

[1] David Lee & Emmanuel Saez, 2012. "Optimal minimum wage policy in competitive labor markets," Journal of Public Economics, vol 96(9-10), pages 739-749.