Section 7 Notes[1]

I. The income taxation problem

- Define the tax in a flexible way using $T(z)$, where z is the income reported by the agent.
- Retention function: $R(z) = z - T(z)$ or $c = z - T(z)$. Measures how much the agent can retain out of total income z.
- Denote transfers to income z with $-T(z)$ so that $-T(0)$ is the transfer to a non-working individual.
- Marginal tax rate: $T'(z)$: individual keeps $1 - T'(z)$ for an additional \$1 of earnings. It measures how much an agent gets taxed out of one additional dollar of income.
- Participation tax rate: $\tau_p = \frac{T(z)-T(0)}{z}$. It's the fraction of income that an agent pays in taxes when they move from 0 income to z.

II. Taxation with no behavioral responses

This ignores labor supply responses to taxation. The agent has utility $u(c)$ such that $u'(c) > 0$ and $u''(c) \leq 0$. Labor does not enter the utility function and is supplied inelastically. The agent consumes everything that is left so that $c = z - T(z)$ holds.

Problem setup:

- Discrete case: N individuals with fixed incomes $z_1 < ... < z_N$. The government then maximizes a utilitarian objective function:

$$SWF = \sum_{i=1}^{N} u(z_i - T(z_i))$$

$$subject \;\; to: \sum_{i=1}^{N} T(z_i) = 0$$

The budget constraint implies that taxes should fund transfers, and the social welfare function means the government is maximizing the sum of after-tax utilities (hence utilitarian: every agent is weighted equally).

[1] These notes are partially based on those of Matteo Paradisi.

- Continuous case: the economy is populated by several agents and their income is distributed according to $h(z)$ with support $[0, \infty]$. The problem of the government is:

$$SWF = \int_0^\infty u(z - T(z))h(z)dz$$
$$subject \;\; to: \int_0^\infty T(z)h(z)dz \geq E$$

Note that E in the budget constraint refers to a revenue requirement.

We solved the problem in the discrete case in the lecture slides. We can solve the problem in the continuous case. We have the following Lagrangian:

$$L = [u(z - T(z))\lambda T(z)]h(z)$$

Where λ is constant aross individuals and measures the value of government revenues in equilibrium. The optimal choice of $T(z)$ comes from the following FOC:

$$\frac{\partial L}{\partial T(z)} = [-u'(z - T(z)) + \lambda]h(z) = 0$$

Then we are left with the condition:

$$u'(z - T(z)) = \lambda$$

λ is a constant and all agents have the same utility function (preferences). Thus, the equilibrium condition implies that consumption is equalized across individuals. This is the consequence of the utilitarian social welfare function and the concavity of utility.

Because there are no behavioral responses, there are no efficiency costs to equalizing consumption among individuals by redistributing from rich to poor. Suppose that $\tilde{z}$ is the average income. What will the marginal tax rate above $\tilde{z}$ be?

III. Optimal linear tax rate with behavioral responses

Back to the discrete case, the government chooses τ to maximize utilitarian social welfare. Note that Z is the total taxable income and it's a function of $(1 - \tau)$, while $z_i = w_i l_i$ is the individual taxable income. (Side question: why does $Z(1 - \tau)$ enter the utility function?). Then the government maximizes:

$$SWF = \sum_i u_i((1 - \tau)w_i l_i + \tau Z(1 - \tau), l_i)$$

The FOC is then:

$$\frac{\partial SWF}{\partial \tau} = \sum_i \frac{\partial u_i}{\partial c}\left[-w_i l_i + Z - \tau \frac{dZ}{d(1 - \tau)}\right] = \sum_i \frac{\partial u_i}{\partial c}\left[-z_i + Z - \tau \frac{dZ}{d(1 - \tau)}\right]$$

What happened to $\frac{\partial u_i}{\partial l_i}$? Envelope Theorem! The agent is already optimizing her own utility with the choice of l_i.
Intuition of Envelope Theorem: because the agent is taking taxes and prices as given and optimizing labor supply the entire time, then the indirect effects on utility sum up to zero. When you have optimized, changing one parameter has only one effect and that is on the objective, and will not have an indirect impact through other parameters.

We then re-arrange the problem after setting $\frac{\partial SWF}{\partial \tau} = 0$:

$$0 = -\sum_i \frac{\partial u_i}{\partial c} z_i + Z \sum_i \frac{\partial u_i}{\partial c} - \left[\tau \frac{dZ}{d(1-\tau)}\right] \sum_i \frac{\partial u_i}{\partial c}$$
$$0 = -\frac{\sum_i \frac{\partial u_i}{\partial c} z_i}{Z \sum_i \frac{\partial u_i}{\partial c}} + \frac{Z \sum_i \frac{\partial u_i}{\partial c}}{Z \sum_i \frac{\partial u_i}{\partial c}} - \left[\tau \frac{dZ}{d(1-\tau)}\right] \frac{\sum_i \frac{\partial u_i}{\partial c}}{Z \sum_i \frac{\partial u_i}{\partial c}}$$

$$1 = \bar{g} + \left[\tau \frac{dZ}{Zd(1-\tau)}\right] = \bar{g} + \left[\tau \frac{dZ(1-\tau)}{Zd(1-\tau)(1-\tau)}\right] = \bar{g} + \frac{\tau}{1-\tau} e_{Z,(1-\tau)}$$

$$\tau^* = \frac{1-\bar{g}}{1-\bar{g}+e_{Z,(1-\tau)}}$$

where $\bar{g} = \frac{\sum_i \frac{\partial u_i}{\partial c} z_i}{Z \sum_i \frac{\partial u_i}{\partial c}}$. This is the weighted marginal utility of person i, weighted by the income of the person, and averaged across individuals. ($\bar{g}$ is 1 when the covariance between income and marginal utility is zero: (1) same income, (2) income distributed randomly).

The optimal tax is decreasing in the elasticity and $\bar{g}$. When income is very elastic to taxes, the government wants to avoid negative effects on revenues and transfers coming from distortions to the labor supply - this is the efficiency component of the formula. To incorporate equity in the formula, $\bar{g}$ is a measure of inequality in the economy and is low when the income distribution is very spread out. The government will therefore want to increase taxes at the optimum when inequality is high.

What happens when the government is Rawlsian rather than utilitarian? A Rawlsian government maximizes the welfare of the poorest individual. This means the government wants to maximize the R component in the linear taxation utility function - we are back at the revenue maximization case where $\bar{g} = 0$.

IV. Optimal top tax rate

In this case, we will not specify a model. We will instead consider how a tax change affects taxation, with the constraint that the sum of all effects is zero in equilibrium. Consider the case where z^* is the income cutoff for the highest marginal tax rate bracket. The government then sets τ for all income above z^*. The average income above z^* is z and depends on the tax rate.

When the tax is raised, individuals with incomes below z^* will not be affected. We have three different effects of the tax on individuals above z^*:

1- **Mechanical effect**: the effect of the tax assuming that labor supply doesn't change:

$$dM = d\tau(z - z^*)$$

2- **Behavioral effect**: describes the reaction of top taxpayers to the higher tax by reducing labor supply, which results in a reduction in revenue:

$$\begin{aligned} dB &= \tau dz = -\tau \frac{dz}{d(1-\tau)} d\tau \\ &= -\frac{\tau}{1-\tau} \frac{1-\tau}{z} \frac{dz}{d(1-\tau)} d\tau \\ &= -\frac{\tau}{1-\tau} \epsilon_{z,1-\tau} z d\tau \end{aligned}$$

3- **Welfare effect**: let $\bar{g}$ be the constant social marginal welfare weight for earners above z^*. The tax changes causes a loss of welfare for top earners equivalent to:

$$dW = -d\tau \bar{g}(z - z^*)$$

The optimal tax: adding all three effects and setting equal to zero:

$$dB + dM + dW = d\tau \left[(1-\bar{g})(z - z^*) - \epsilon_{z,1-\tau} \frac{\tau}{1-\tau} z \right] = 0$$

We can rearrange:

$$\begin{aligned} 0 &= (1-\bar{g}) \frac{z - z^*}{z} - \epsilon_{z,1-\tau} \frac{\tau}{1-\tau} \\ \frac{z}{z - z^*} \epsilon \frac{\tau}{1-\tau} &= (1-\bar{g}) \\ a\epsilon\tau + \tau(1-\bar{g}) &= (1-\bar{g}) \\ \tau^* &= \frac{1-\bar{g}}{1-\bar{g}+a\epsilon_{z,1-\tau}} \end{aligned}$$

where $a = \frac{z}{z-z^*}$, which measures the thickness of the upper tail of the income distribution.

What does the equation say?

1- The optimal tax is decreasing in social marginal welfare weight of top earners ($\bar{g}$): the more the government cares about high income individuals the less they will be taxed. In the limit case where the government doesn't place any weight on the marginal consumption of

the top earner, we are at the revenue maximizing top rate $\frac{1}{1+a\epsilon}$.
2- The optimal tax is decreasing in the elasticity of labor supply: higher elasticity implies higher efficiency costs. This is the *uncompensated* elasticity.
3- The optimal tax is decreasing in the thinness of the tail: the government imposes a lower tax when income is more concentrated at z^* rather than the rest of the upper tail.

For extra reading: Piketty, Thomas, and Emmanuel Saez. "Optimal Taxation of Labor Income." Handbook of Public Economics. 2013.