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**SECTION 9**

# I. Health insurance

## A. Heterogeneity in risk across individuals: symmetric information

Insurance companies and individuals can observe types $q_H$ and $q_S$. Then insurance companies will set policies with two actuarially fair prices: $p_S$ for the sickly type and $p_H$ for the healthy type. That is, the insurance problem is the same as in the case where risk is homogenous across individuals and the individual maximizes expected utility by choosing a level of premium $p$. Except in this case, we have two types of individuals with two maximization problems corresponding to their types, probability of getting sick, income, premium and payout. Consider the sickly individual's maximization problem:

$$EU_S = (1 - q_S)U(W - p_S) + q_S U(W - d_S - p_S + b_S)$$

and where perfect competition means the insurance company sets

$$EP_S = p_S - q_S b_S = 0$$

Therefore:

$$EU_S = (1 - q_S)U(W - p_S) + q_S U(W - d_S - p_S + p_S/q_S)$$

Let's maximize the expected utility for the sickly individual:

$$dEU_S/dp_S = -(1 - q_S)U'(W - p_S) + q_S[-1 + 1/q_S]U'(W - d_S - p_S + p_S/q_S) = 0$$

$$U'(W - p_S) = U'(W - d_S - p_S + p_S/q_S)$$

Then $p_S = d_S q_S$, implying the sickly person obtains consumption in both states equal to

$$W - d_S q_S$$

Note we can do the same for the healthy type with subscript $H$. The key to this problem is that neither type can pretend to be the other type because the insurer has complete information about everyone's type.

## B. Heterogeneity in risk across individuals: asymmetric information

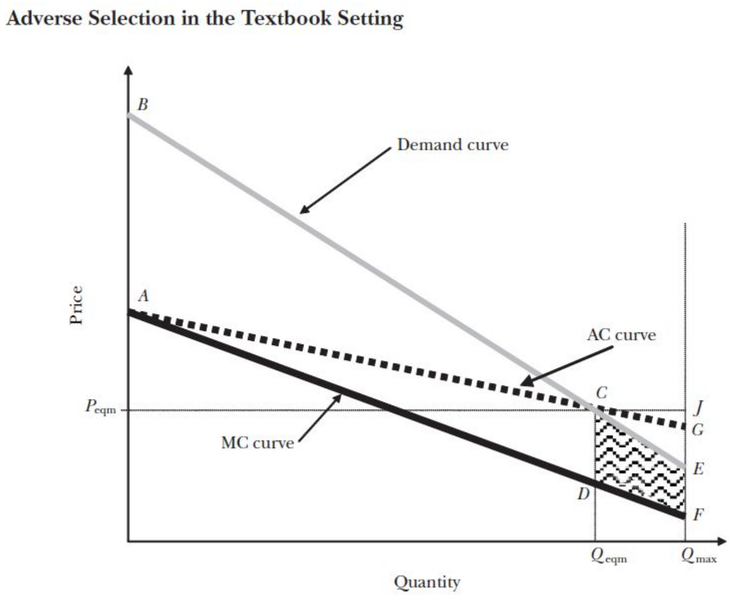
Insurance companies cannot observe $q_H$ vs. $q_S$ types but individuals know their own types.

Two equilibrium possibilities:
(1) Pooling equilibrium: Insurance companies offer a contract based on average risk [good

deal for sickly, mediocre deal for healthy but better than no insurance]
(2) Separating equilibrium: Insurance companies offer two contracts: one expensive contract with full insurance for the sickly, one cheap contract with partial insurance for the healthy: each type self-select into its contract → Outcome not efficient as healthy are under-insured.

If insurance companies charge $p_S = q_S d$ and $p_H = q_H d$, then all the sickly types would want to buy the healthy insurance which is cheaper. However, since the healthy insurance is priced at an actuarially fair price for the healthy, when the sickly enter the insurance market for the healthy the insurance companies make losses. This is adverse selection.

Figure I: Adverse selection in the insurance market

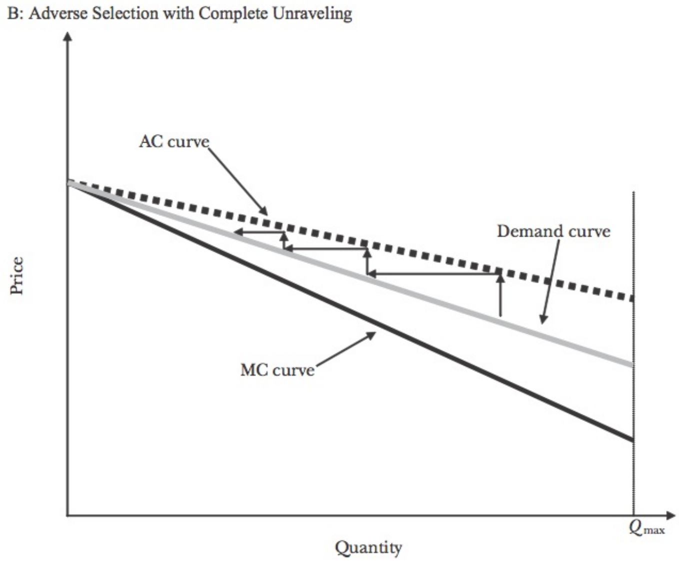



Source: Einav and Finkelstein (2011 JEP)

Complete unraveling: the market can unravel in a death spiral. Insurance offered at an average fair price $\implies$ bad deal for low risk people if the high risk (sickly) take up the health insurance offered to the healthy $\implies$ low risk people leave and only high risk remain $\implies$ insurers make losses $\implies$ insurers raise the price to break even $\implies$ only very high risk people buy in $\implies$ this continues until no insurance contract is offered even if everyone wants full actuarially fair insurance

Market unravels if no one is willing to pay the pooled cost of those with higher demand (and thus likely to be higher risk):

Figure II: Complete unraveling




Source: Einav and Finkelstein (2011 JEP)

# II. Optimal social insurance[1]

We will work through a version of the Chetty-Baily model on optimal social insurance. Suppose that when an individual is employed, she receives a wage $w_H$, and when she is unemployed, she receives a wage $w_L$, with $w_H > w_L$. Let $A$ denote wealth, $c_H$ denote consumption in the employed state, and $c_L$ denote consumption in the low state.

Assume the agent is initially unemployed. She controls the probability $p$ of becoming employed by exerting effort $e$ at a cost $\psi(e)$. For simplicity, assume that the probability of becoming employed is normalized to $p(e) = e$.

Finally, the government sets up an unemployment insurance system. Those who are unemployed receive a constant benefit $b$, which is financed by a lump sum tax $t$ on individuals who are employed.

**A. The individual's utility function is given by:**

$$U(e, b, t) = eu(A + w_H - t) + (1 - e)u(A + w_L + b) - \psi(e)$$

Because the probability of becoming employed is normalized to $p(e) = e$, we see that this is simply an expected utility. The first term, $eu(A+w_H-t)$ is the probability of being employed, $e$, times the utility in the employed state, $u(A + w_H - t)$. Furthermore, $(1 - e)u(A + w_L + b)$ is the probability of being unemployed times the utility in that state. Finally, the individual faces a cost $\psi(e)$ from exerting effort.

**B. Takes $b$ and $t$ as given, then we can find the first order condition for the optimal amount of effort:**

[1] These notes draw on previous notes from Xavier Jaravel.

Taking $b$ and $t$ as given, the agent solves $\max_e eu(A+w_H-t)+(1-e)u(A+w_L+b)-\psi(e)$. The FOC is

$$u(A+w_H-t)-u(A+w_L+b)=\psi'(e) \implies u(c_H)-u(c_L)=\psi'(e).$$

**C. Now write down the government budget constraint in terms of $e$, $b$, and $t$:**

The government collects taxes only from those in the employed state and pays benefits to those in the unemployed state. Since there is a probability $e$ of being in the employed state, we must have

$$et=(1-e)b \implies t=\left(\frac{1-e}{e}\right)b.$$

**D. The government knows the individual's first order condition, then the government's maximization problem can be expressed as follows:**

$$\max_b e(b)u(A+w_H-t(b))+(1-e(b))u(A+w_L+b)-\psi(e(b))$$

This set up is very similar to the individual's maximization, but the government takes into account that the individual's effort $e$ depends on the benefits $b$ and taxes $t$. Because the government faces a budget constraint, we can write both the effort and the taxes as a function of the benefit and maximize $b$.

**E. We will solve the government's problem in several steps. First, use the fact shown above that $t=\left(\frac{1-e}{e}\right)b$ to show that $t'(b)=\frac{1-e}{e}\left(1+\frac{\epsilon_{1-e,b}}{e}\right)$:**

Then, we have

$$t'(b)=\frac{d\left(\frac{1-e(b)}{e(b)}b\right)}{db}=\frac{1-e(b)}{e(b)}+\frac{-\frac{de}{db}e(b)-(1-e(b))\frac{de}{db}}{e^2}b=\frac{1-e(b)}{e(b)}-\frac{b}{e^2}\frac{de}{db}=\frac{1-e}{e}\left(1+\frac{\epsilon_{1-e,b}}{e}\right).$$

**F. Now find the first order condition for the government:**

There are two ways to do this: with and without the envelope theorem. We will first solve it without the envelope theorem. Taking the derivative, we get

$$\begin{aligned} e(b)u'(A+w_H-t(b))(-t'(b))+e'(b)u(A+w_H-t(b)) \\ +(1-e(b))u'(A+w_L+b)-e'(b)u(A+w_L+b)-\psi(e(b))e'(b)=0 \end{aligned}$$

,

$$\begin{aligned} e(b)u'(A+w_H-t(b))(-t'(b))+(1-e(b))u'(A+w_L+b) \\ +e'(b)u(A+w_H-t(b))-e'(b)u(A+w_L+b)-\psi(e(b))e'(b)=0 \implies \\ e(b)u'(A+w_H-t(b))(-t'(b))+(1-e(b))u'(A+w_L+b) \\ +e'(b)(u(A+w_H-t(b))-u(A+w_L+b)-\psi(e(b)))=0 \implies \\ e(b)u'(A+w_H-t(b))(-t'(b))+(1-e(b))u'(A+w_L+b)=0 \end{aligned}$$

Note that everything in blue is zero since it's simply the individual's first order condition. The envelope theorem is just a shortcut to this conclusion. It says that because the individual has already maximized, the first order condition for the government does not need to take into account behavioral responses to marginal policy changes. In math, this means that we can just skip to

$$e(b)u'(A + w_H - t(b))(-t'(b)) + (1 - e(b))u'(A + w_L + b) \quad = \quad 0$$

Solving with invoking the envelope theorem directly:

In maximizing, we did not need to treat $e$ as depending on $b$. This is due to the envelope theorem. The individual has already maximized effort taking $b$ as given, and the theorem implies that we only need to worry about the direct effects of changing $b$, not the indirect effects through $e$. Note that we still have to take into account $t$'s dependence on $b$ since $t$ is the government's choice. Either way, we get

$$\begin{aligned}
eu'(A + w_H - t(b))(-t'(b)) + (1 - e(b))u'(A + w_L + b) &= 0 \Longrightarrow \\
eu'(c_H)\left(-\frac{1-e}{e}\left[1 + \frac{\epsilon_{1-e,b}}{e}\right]\right) + (1-e)u'(c_L) &= 0 \Longrightarrow \\
eu'(c_H)\left(\frac{1-e}{e}(1 + \epsilon_{1-e,b}e)\right) &= (1-e)u'(c_L) \Longrightarrow \\
(1-e)(u'(c_H) - u'(c_L)) + (1-e)u'(c_H)\frac{\epsilon_{1-e,b}}{e} &= 0 \Longrightarrow \\
\frac{u'(c_L) - u'(c_H)}{u'(c_H)} &= \frac{\epsilon_{1-e,b}}{e}
\end{aligned}$$

**G. The intuition:**

The left-hand side can be thought of as the benefit from unemployment insurance. $\frac{u'(c_L)-u'(c_H)}{u'(c_H)}$ represents the consumption smoothing benefits to be gained. The right-hand side is the cost. $\frac{\epsilon_{1-e,b}}{e}$ represents the moral hazard costs as it is the elasticity of being unemployed with respect to benefits.