

# Lectures 2 and 3: Optimal Income Taxation

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# GOALS OF THESE LECTURES

- 1) Take a look at **actual taxes and transfers** in the U.S.
- 2) Understand the **core optimal income tax model**: linear and nonlinear taxes in the Saez (2001) framework.

General method, intuitive, sufficient statistics.

- 3) Introduce the mechanism design approach of **Mirrlees (1971)**.

Incentive compatibility, optimal control.

With and without income effects.

- 4) Extensions: Migration and rent-seeking

- 5) Should commodity taxes be used in addition to income taxes?

Atkinson-Stiglitz Theorem

# TAXATION AND REDISTRIBUTION

**Key question:** Should government reduce inequality using taxes and transfers?

1) Governments use **taxes** to raise revenue

2) This revenue funds **transfer** programs:

a) Universal Transfers: Education, Health Care (only 65+ in the US), Retirement and Disability

b) Means-tested Transfers: In-kind (e.g., public housing, nutrition, Medicaid in the US) and cash

Modern governments raise large fraction of GDP in taxes (30-45%) and spend significant fraction of GDP on transfers

This lecture follows Piketty and Saez '13 **handbook chapter**

## GOAL: TAKE A LOOK AT ACTUAL TAX SYSTEM

Sometimes you are an optimal tax theorist and don't know the actual top tax rates – it's weird.

You need to know institutional details. It's not boring. It's crucial.

You should not try to capture all institutional details in your models. But unless you know them, you cannot argue they are second-order. (Sometimes the devil is in the detail, sometimes not).

The tax system reflects

- i) social judgements made by people and policy makers and
- ii) lobbying, political economy, interest groups.

Understand the implicit social judgements behind the tax system.

Question them! Which constraints are truly “irremovable”?

## FACTS ON US TAXES AND TRANSFERS

**References:** Comprehensive description in Gruber undergrad textbook (taxes/transfers) and Slemrod-Bakija (taxes)

<http://www.taxpolicycenter.org/taxfacts/>

**A) Taxes:** (1) individual income tax (fed+state), (2) payroll taxes on earnings (fed, funds Social Security+Medicare), (3) corporate income tax (fed+state), (4) sales taxes (state)+excise taxes (state+fed), (5) property taxes (state)

**B) Means-tested Transfers:** (1) refundable tax credits (fed), (2) in-kind transfers (fed+state): Medicaid, public housing, nutrition (SNAP), education (3) cash welfare: TANF for single parents (fed+state), SSI for old/disabled (fed)

## FEDERAL US INCOME TAX

US income tax assessed on **annual family** income (not individual) [most other OECD countries have shifted to individual assessment]

Sum all cash income sources from family members (both from labor and capital income sources) = called **Adjusted Gross Income (AGI)**

Main exclusions: fringe benefits (health insurance, pension contributions and returns), imputed rent of homeowners, undistributed corporate profits, interest from state+local bonds, unrealized capital gains

⇒ AGI base is only 70% of factor national income (earned by some factor at some point).

## FEDERAL US INCOME TAX

Taxable income = AGI - personal exemptions - deduction

personal exemptions = Used to be \$4K \* # family members (in 2016/17).  
Removed in TCJA 2018.

deduction is max of standard deduction or itemized deductions

Standard deduction is a fixed amount depending on family structure (\$24K for couple, \$12K for single in 2018 (it basically doubled when personal exemption abolished))

Itemized deductions: (a) state and local taxes paid (new: up to \$10K now), (b) mortgage interest payments, (c) charitable giving, various small other items

[about 10% of AGI lost through itemized deductions, called **tax expenditures**]

## FEDERAL US INCOME TAX: TAX BRACKETS

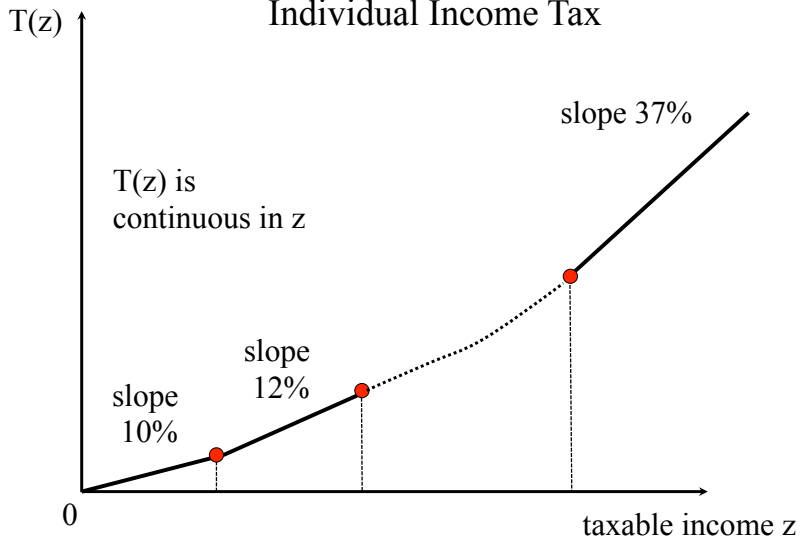
Tax  $T(z)$  is piecewise linear and continuous function of taxable income  $z$  with constant marginal tax rates (MTR)  $T'(z)$  by brackets

In 2018+, 7 brackets with MTR 10%,12%,22%,24%,32%,35%, 37% (top bracket for  $z$  above \$600K), indexed on price inflation

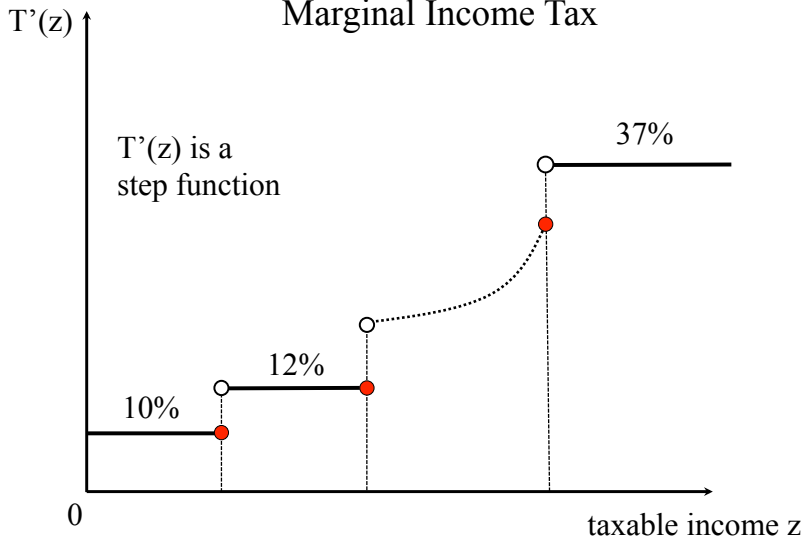
Lower preferential rates (up to a max of 20%) apply to dividends (since 2003) and realized capital gains [in part to offset double taxation of corporate profits]

Tax rates change frequently over time. Top MTRs have declined drastically since 1960s (as in many OECD countries)

# Individual Income Tax

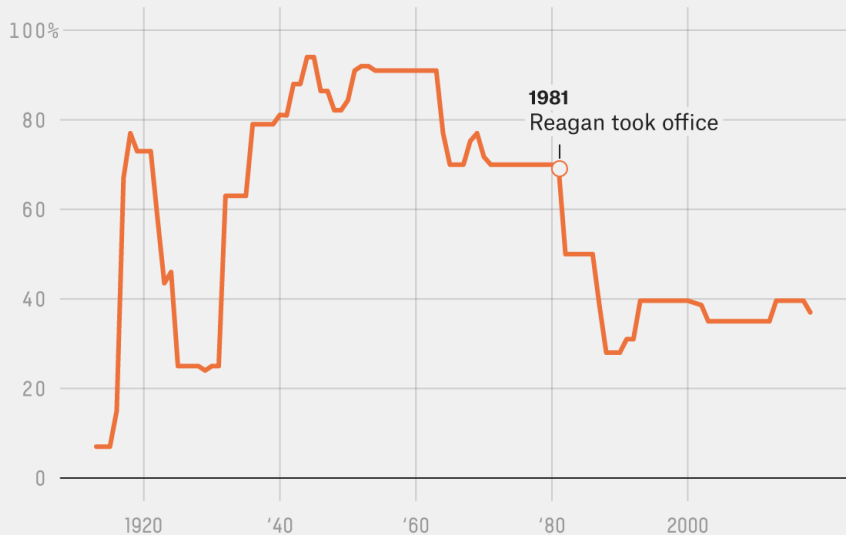


# Marginal Income Tax



# Historically, a 70 percent marginal tax rate is not unusual

The top marginal income tax rates from 1913 to 2018



In practice...

Form **1040** U.S. Individual Income Tax Return **2010** (99) IRS Use Only - Do not write or staple in this space.

Name, Address, and SSN  
**PRINT**  
 For the year Jan. 1-Dec. 31, 2010, or other tax year beginning \_\_\_\_\_, 2010, ending \_\_\_\_\_ 20  
 OMB No. 1545-0074  
 Your first name and initial  
**BARACK H.** Last name  
**OBAMA** Your social security number  
 If a joint return, spouse's first name and initial  
**MICHELLE L.** Last name  
**OBAMA** Spouse's social security number  
**CLEARLY**  
 Home address (number and street). If you have a P.O. box, see instructions. Apt. no.  
**1600 PENNSYLVANIA AVENUE, NW** Make sure the SSN(s) above  
 City, town or post office, state, and ZIP code. **▲** and on the 5c are correct.  
**WASHINGTON, DC 20500** Checking a box below will not change your tax or refund.

Presidential Election Campaign  Check here if you, or your spouse if filing jointly, want \$3 to go to this fund  You  Spouse

Filing Status  
 1  Single  
 2  Married filing jointly (even if only one had income)  
 3  Married filing separately. Enter spouse's SSN above  
 4  Head of household (with qualifying person). If the qualifying person is a child but not your dependent, enter this child's name here.   
 5  Qualifying widow(er) with dependent child

Exemptions  
 6a  Yourself. If someone can claim you as a dependent, do not check box 6a  
 6b  Spouse  
 c Dependents:  
 (1) First name Last name (2) Dependent's social security number (3) Dependent's relationship to you (4) If (3) is "child" (or age 17 or older), check box for child tax credit  

<b>MALIA A OBAMA</b>			<b>DAUGHTER</b>	<input checked="" type="checkbox"/>
<b>NATASHA M OBAMA</b>			<b>DAUGHTER</b>	<input checked="" type="checkbox"/>

 Boxes checked on 6a and 6b  
 No. of children on file who:  
 • lived with you  
 • did not live with you due to divorce or separation (see instructions)  
 Dependents on 6c not entered above  
 Add numbers on lines above **4**

Income  
 7 Wages, salaries, tips, etc. Attach Form(s) W-2 7 **395,188.**  
 8a Taxable interest. Attach Schedule B if required 8a **8,066.**  
 b Tax-exempt interest. Do not include on line 8a 8b  
 9a Ordinary dividends. Attach Schedule B if required 9a **9,997.**  
 b Qualified dividends 9b **2,159.**  
 10 Taxable refunds, credits, or offsets of state and local income taxes **STMT 2 STMT 3** 10 **1,151.**  
 11 Alimony received 11  
 12 Business income or (loss). Attach Schedule C or C-EZ 12 **1,382,889.**  
 13 Capital gain or (loss). Attach Schedule D if required. If not required, check here  13 **-3,000.**

## 2018 US Personal Income Tax Code

Rate	Individuals	Married Filing Jointly
10%	Up to \$9,525	Up to \$19,050
12%	\$9,526 to \$38,700	\$19,051 to \$77,400
22%	\$38,701 to \$82,500	\$77,401 to \$165,000
24%	\$82,501 to \$157,500	\$165,001 to \$315,000
32%	\$157,501 to \$200,000	\$315,001 to \$400,000
35%	\$200,001 to \$500,000	\$400,001 to \$600,000
37%	over \$500,000	over \$600,000

## FEDERAL US INCOME TAX: AMT AND CREDITS

**Alternative minimum tax (AMT)** is a parallel tax system (quasi flat tax at 28%) with fewer deductions: actual tax =  $\max(T(z), AMT)$  (hits < 1% of taxpayers in 2018+)

**Tax credits:** Additional reduction in taxes

- (1) **Non refundable** (cannot reduce taxes below zero): foreign tax credit, child care expenses, education credits, energy credits
- (2) **Refundable** (can reduce taxes below zero, i.e., be net transfers): EITC (earned income tax credit, up to \$3.5K, \$5.7K, \$6.5K for working families with 1, 2, 3+ kids), Child Tax Credit (\$2K per kid, partly refundable)

## FEDERAL US INCOME TAX: TAX FILING

Taxes on year  $t$  earnings are withheld on paychecks during year  $t$  (pay-as-you-earn) (Why?)

Income tax return filed in January–April 15, year  $t + 1$  [filers use either software or tax preparers, **huge** private industry]

Most tax filers get a tax refund as withholdings  $>$  net taxes owed

Payers (employers, banks, etc.) send income information to IRS (US Tax administration) (3rd party reporting)

Third party reporting + withholding at source is key for successful enforcement

## MAIN MEANS-TESTED TRANSFER PROGRAMS

1) **Traditional transfers:** managed by welfare agencies, paid on monthly basis, high stigma and take-up costs  $\Rightarrow$  low take-up rates

Main programs: Medicaid (health insurance for low incomes), Supplemental Nutritional Assistance Program (SNAP, former food stamps), public housing, Temporary Assistance to Needy Families (TANF, traditional welfare), Supplemental Security Income (SSI) (aged+disabled)

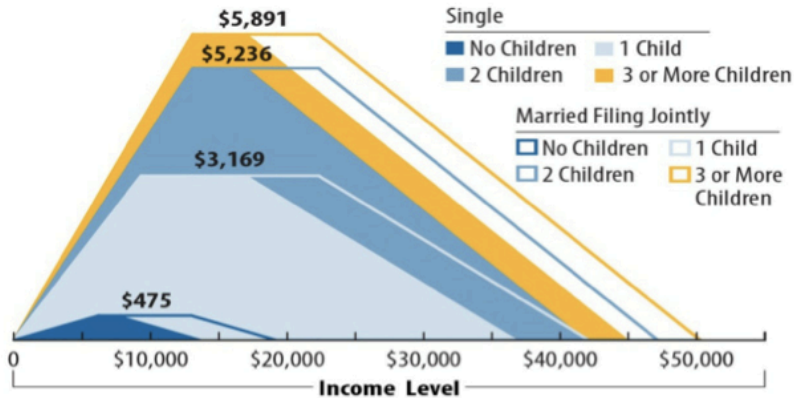
2) **Refundable income tax credits:** managed by tax administration, paid as an annual lumpsum in year  $t + 1$ , low stigma and take-up cost  $\Rightarrow$  high take-up rates

Main programs: EITC and Child Tax Credit [large expansion since the 1990s] for low income working families with children

$\rightarrow$  move has been from “support the very poor” to “support working low-income.”

Figure 1

EITC refunds by family size and income (CBPP 2013)



Source: Center on Budget and Policy Priorities.

## BOTTOM LINE ON ACTUAL TAXES/TRANSFERS

- 1) Based on current income, family situation, and disability (retirement) status ⇒ Strong link with **current ability to pay**
- 2) Some allowances made to reward / encourage certain behaviors: charitable giving, home ownership, savings, energy conservation, and more recently work (refundable tax credits such as EITC)

Do you think this is the role of the tax system?

- 3) Provisions pile up overtime making tax/transfer system more and more complex until significant simplifying reform happens (such as US Tax Reform Act of 1986, or TCJA 2018)

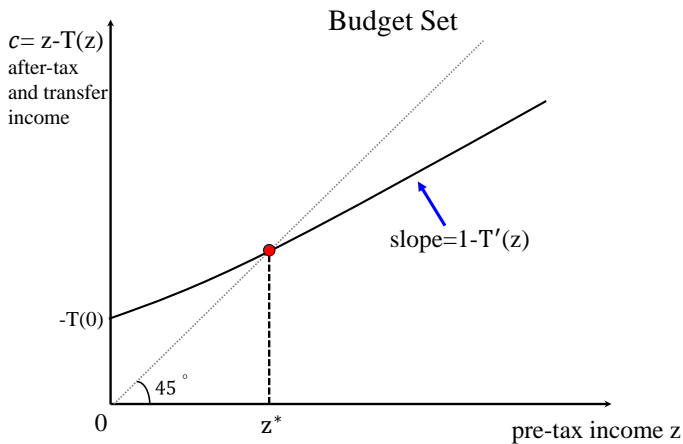
Motto: any vested interest you create will be impossible to remove.

## KEY CONCEPTS FOR TAXES/TRANSFERS

- 1) Transfer benefit with zero earnings  $-T(0)$  [sometimes called demogrant or lumpsum grant] Fancy word: Universal Basic Income (UBI)
- 2) Marginal tax rate (or phasing-out rate)  $T'(z)$ : individual keeps  $1 - T'(z)$  for an additional \$1 of earnings (intensive labor supply response)
- 3) Participation tax rate  $\tau_p = [T(z) - T(0)]/z$ : individual keeps fraction  $1 - \tau_p$  of earnings when moving from zero earnings to earnings  $z$  (extensive labor supply response):

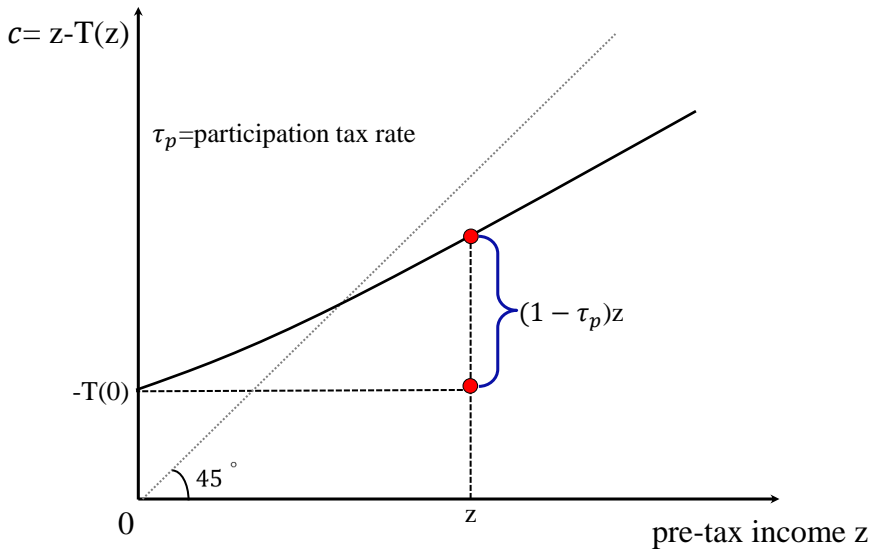
$$z - T(z) = -T(0) + z - [T(z) - T(0)] = -T(0) + z \cdot (1 - \tau_p)$$

- 4) Break-even earnings point  $z^*$ : point at which  $T(z^*) = 0$



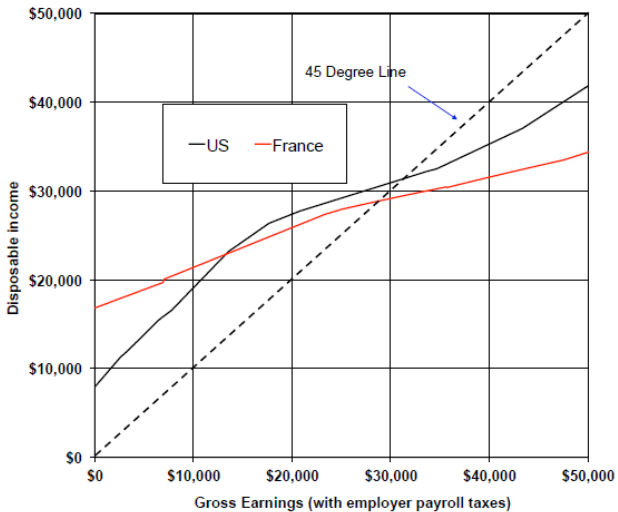
If line is steeper is that more or less redistribution?

What is perfect redistribution? What is no redistribution?



### US Tax/Transfer System, single parent with 2 children, 2009





Source: Piketty, Thomas, and Emmanuel Saez (2012)

## OPTIMAL TAXATION: SIMPLE MODEL WITH NO BEHAVIORAL RESPONSES

Utility  $u(c)$  strictly increasing and concave

Same for everybody where  $c$  is after tax income.

Income is  $z$  and is fixed for each individual,  $c = z - T(z)$  where  $T(z)$  is tax on  $z$ .  $z$  has density distribution  $h(z)$

Government maximizes **Utilitarian** objective:

$$\int_0^{\infty} u(z - T(z))h(z)dz$$

subject to **budget constraint**  $\int T(z)h(z)dz \geq E$  (multiplier  $\lambda$ )

## SIMPLE MODEL WITH NO BEHAVIORAL RESPONSES

Form lagrangian:  $L = [u(z - T(z)) + \lambda \cdot T(z)] \cdot h(z)$

First order condition (FOC) in  $T(z)$ :

$$0 = \frac{\partial L}{\partial T(z)} = [-u'(z - T(z)) + \lambda] \cdot h(z) \Rightarrow u'(z - T(z)) = \lambda$$

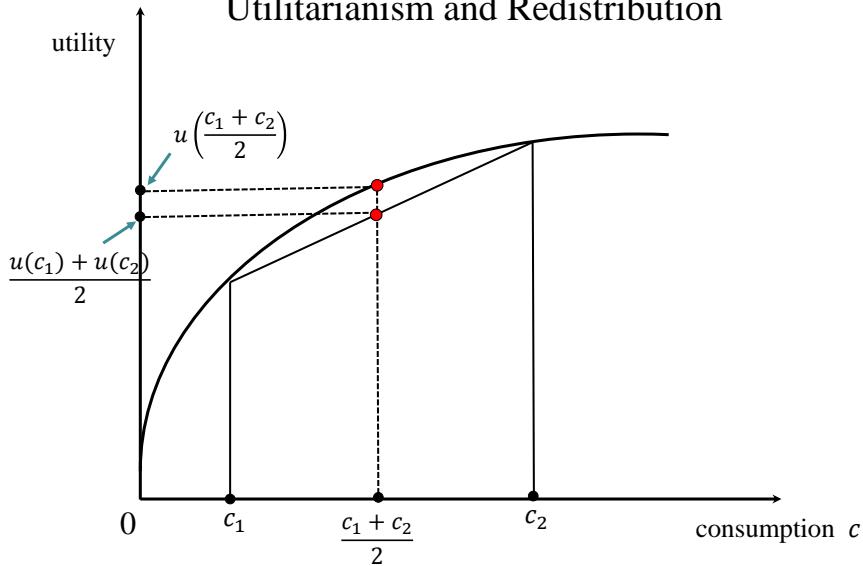
$\Rightarrow z - T(z) = \text{constant for all } z.$

$\Rightarrow c = \bar{z} - E$  where  $\bar{z} = \int zh(z)dz$  average income.

100% marginal tax rate. Perfect equalization of after-tax income.

Utilitarianism with decreasing marginal utility leads to perfect egalitarianism [Edgeworth, 1897]

## Utilitarianism and Redistribution



## ISSUES WITH SIMPLE MODEL

1) **No behavioral responses:** Obvious missing piece: 100% redistribution would destroy incentives to work and thus the assumption that  $z$  is exogenous is unrealistic

⇒ Optimal income tax theory incorporates behavioral responses (Mirrlees REStud '71): **equity-efficiency trade-off**

2) **Issue with Utilitarianism:** Even absent behavioral responses, many people would object to 100% redistribution [perceived as confiscatory]

⇒ Citizens' views on fairness impose **bounds** on redistribution.

The issue is the restricted nature of social preferences that can be captured by most social welfare functions.

We will discuss preferences for redistribution in another lecture! For now we remain agnostic about the " $g_i$ ".

# MIRRLEES OPTIMAL INCOME TAX MODEL

We will solve the Mirrleesian model later. For now, let's look at the spirit of optimal tax evolution.

1) **Standard labor supply model:** Individual maximizes  $u(c, l)$  subject to  $c = wl - T(wl)$  where  $c$  consumption,  $l$  labor supply,  $w$  wage rate,  $T(\cdot)$  nonlinear income tax  $\Rightarrow$  taxes affect labor supply

2) **Individuals differ in ability  $w$ , private information  $w$**  distributed with density  $f(w)$ .

3) **Govt social welfare maximization:** Govt maximizes

$$SWF = \int G(u(c, l))f(w)dw$$

( $G(\cdot)$   $\uparrow$  concave) subject to

(a) budget constraint  $\int T(wl)f(w)dw \geq E$  (multiplier  $\lambda$ )

(b) individuals' labor supply  $l$  depends on  $T(\cdot)$

## MIRRLEES MODEL RESULTS

Optimal income tax trades-off redistribution and efficiency (as tax based on  $w$  only not feasible)

$\Rightarrow T(.) < 0$  at bottom (transfer) and  $T(.) > 0$  further up (tax) [full integration of taxes/transfers]

Mirrlees formulas complex, only a couple fairly general results:

- 1)  $0 \leq T'(\cdot) \leq 1$ ,  $T'(\cdot) \geq 0$  is non-trivial (rules out EITC) [Seade '77]
- 2) Marginal tax rate  $T'(\cdot)$  should be zero at the top (if skill distribution bounded) [Sadka '76-Seade '77]
- 3) If everybody works and lowest  $w/l > 0$ ,  $T'(\cdot) = 0$  at bottom

## HISTORY: BEYOND MIRRLEES

Mirrlees '71 had a huge impact on information economics: models with asymmetric information in contract theory

Discrete 2-type version of Mirrlees model developed by Stiglitz JpubE '82 with individual FOC replaced by Incentive Compatibility constraint [high type should not mimick low type]

Till late 1990s, Mirrlees results not closely connected to empirical tax studies and little impact on tax policy recommendations

Since late 1990s, Diamond AER'98, Piketty '97, Saez ReStud '01 have connected Mirrlees model to practical tax policy / empirical tax studies

[new approach summarized in Diamond-Saez JEP'11 and Piketty-Saez Handbook'13]

## INTENSIVE LABOR SUPPLY CONCEPTS

$$\max_{c, z} u(c, z) \text{ subject to } c = z \cdot (1 - \tau) + R$$

Imagine a linearized budget constraint:  $R$  is virtual income (why virtual?) and  $\tau$  marginal tax rate.

FOC in  $c, z \Rightarrow (1 - \tau)u_c + u_z = 0 \Rightarrow$  Marshallian labor supply  
 $z = z(1 - \tau, R)$

$$\text{Uncompensated elasticity } \epsilon^u = \frac{(1 - \tau)}{z} \frac{\partial z}{\partial (1 - \tau)}$$

$$\text{Income effects } \eta = (1 - \tau) \frac{\partial z}{\partial R} \leq 0$$

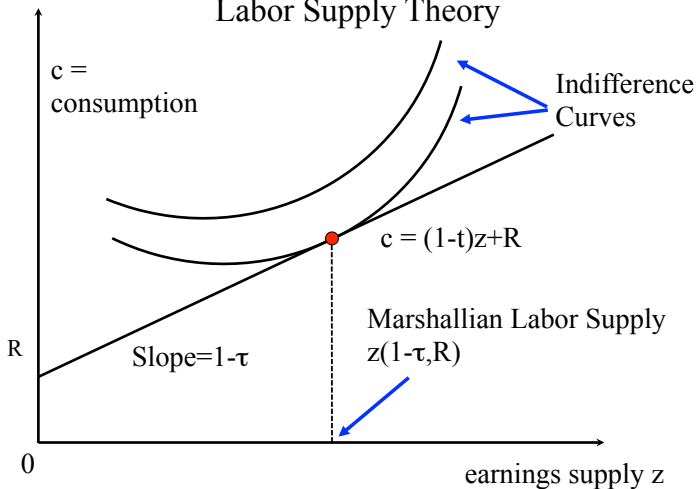
## INTENSIVE LABOR SUPPLY CONCEPTS (II)

Substitution effects: Hicksian labor supply:  $z^c(1 - \tau, u)$  minimizes cost needed to reach  $u$  given slope  $1 - \tau \Rightarrow$

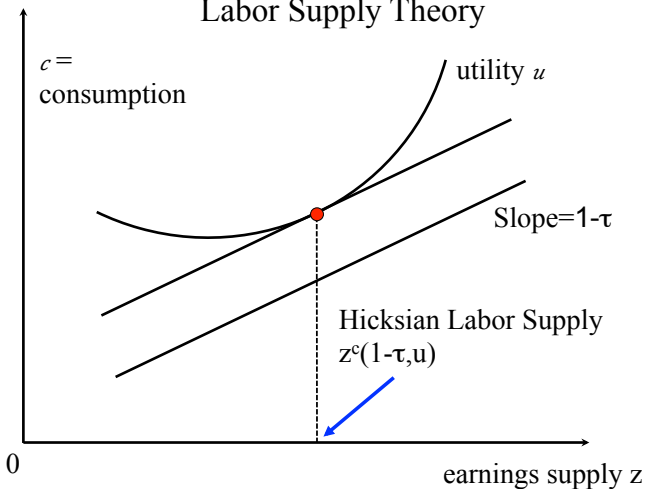
$$\text{Compensated elasticity } \varepsilon^c = \frac{(1 - \tau)}{z} \frac{\partial z^c}{\partial (1 - \tau)} > 0$$

$$\text{Slutsky equation } \frac{\partial z}{\partial (1 - \tau)} = \frac{\partial z^c}{\partial (1 - \tau)} + z \frac{\partial z}{\partial R} \Rightarrow \varepsilon^u = \varepsilon^c + \eta$$

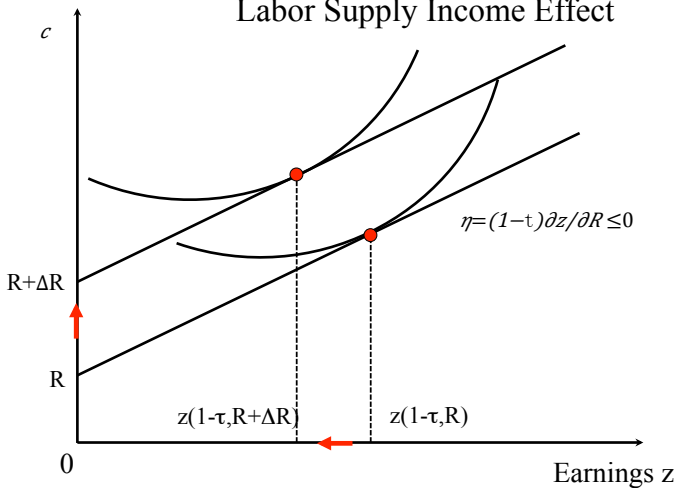
# Labor Supply Theory



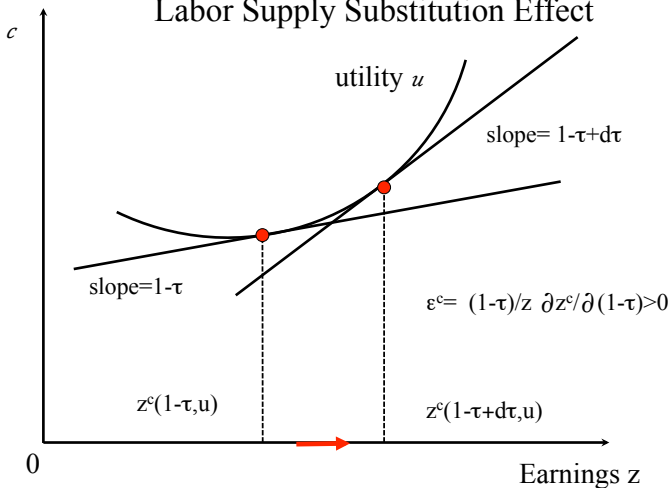
## Labor Supply Theory



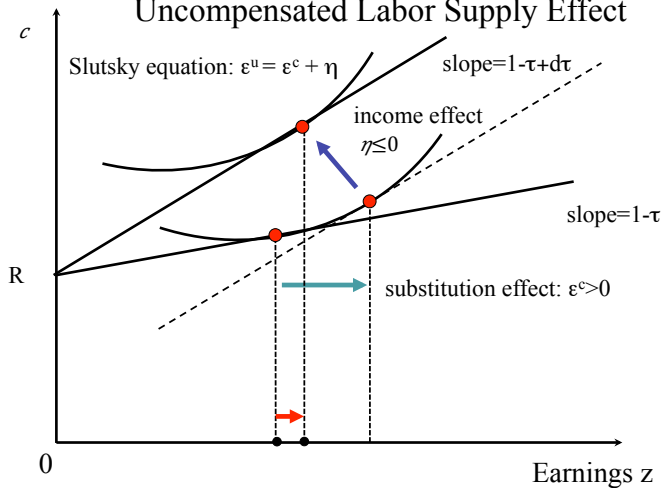
# Labor Supply Income Effect



## Labor Supply Substitution Effect



# Uncompensated Labor Supply Effect



## Labor Supply Effects of Taxes and Transfers

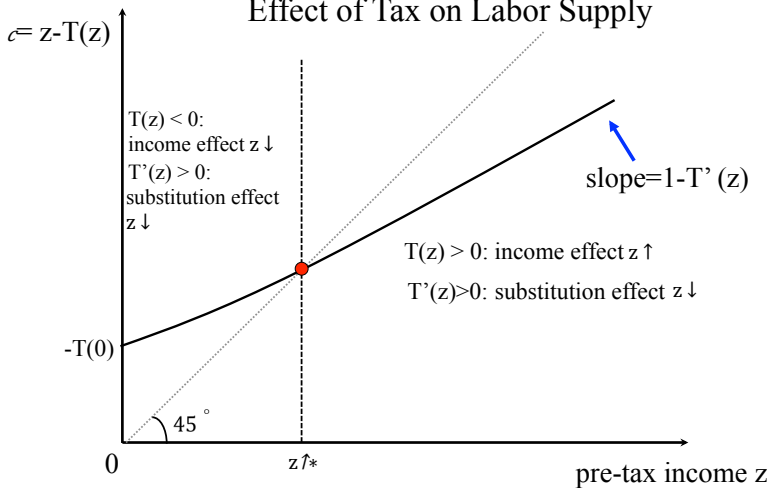
Taxes and transfers change the slope  $1 - T'(z)$  of the budget constraint and net disposable income  $z - T(z)$  (relative to the no tax situation where  $c = z$ )

Positive MTR  $T'(z) > 0$  reduces labor supply through substitution effects

Net transfer ( $T(z) < 0$ ) reduces labor supply through income effects

Net tax ( $T(z) > 0$ ) increases labor supply through income effects

## Effect of Tax on Labor Supply



## WELFARE EFFECT OF SMALL TAX REFORM

Indirect utility:  $V(1 - \tau, R) = \max_z u((1 - \tau)z + R, z)$  where  $R$  is virtual income intercept

Small tax reform:  $d\tau$  and  $dR$ :

$$dV = u_c \cdot [-zd\tau + dR] + dz \cdot [(1 - \tau)u_c + u_z] = u_c \cdot [-zd\tau + dR]$$

**Envelope theorem:** no effect of  $dz$  on  $V$  because  $z$  is already chosen to maximize utility ( $(1 - \tau)u_c + u_z = 0$ )

$[-zd\tau + dR]$  is the mechanical change in disposable income due to tax reform

Welfare impact of a small tax reform is given by  $u_c$  times the money metric mechanical change in tax

## WELFARE EFFECT OF SMALL TAX REFORM (II)

!! Remains true of any nonlinear tax system  $T(z)$

Just need to look at  $dT(z)$ , mechanical change in taxes, or  $dT_i$  for agent  $i$ .

$dV_i$  = Welfare impact is  $-u_c dT(z_i)$ .

When is the welfare impact not just the mechanical change in disposable income?

**Envelope Theorem:** For a constrained problem

$$V(\theta) = \max_x F(x, \theta) \quad \text{s.t.} \quad c \geq G(x, \theta)$$

$$V'(\theta) = \frac{\partial F}{\partial \theta}(x^*(\theta), \theta) - \lambda^*(\theta) \frac{\partial G}{\partial \theta}(x^*(\theta), \theta)$$

## SOCIAL WELFARE FUNCTIONS (SWF)

Welfarism = social welfare based solely on individual utilities

Any other social objective will lead to Pareto dominated outcomes in some circumstances (Kaplou and Shavell JPE'01) Why?

Most widely used welfarist SWF:

1) Utilitarian:  $SWF = \int_i u^i$

2) Rawlsian (also called Maxi-Min):  $SWF = \min_i u^i$

3)  $SWF = \int_i G(u^i)$  with  $G(\cdot) \uparrow$  and concave, e.g.,  $G(u) = u^{1-\gamma}/(1-\gamma)$   
(Utilitarian is  $\gamma = 0$ , Rawlsian is  $\gamma = \infty$ )

4) General Pareto weights:  $SWF = \int_i \mu_i \cdot u^i$  with  $\mu_i \geq 0$  exogenously given

## SOCIAL MARGINAL WELFARE WEIGHTS

Key sufficient statistics in optimal tax formulas are **Social Marginal Welfare Weights** for each individual:

Social Marginal Welfare Weight on individual  $i$  is  $g_i = G'(u^i)u_c^i / \lambda$  ( $\lambda$  multiplier of govt budget constraint) measures \$ value for govt of giving \$1 extra to person  $i$

No income effects  $\Rightarrow \int_i g_i = 1$ : giving \$1 to all costs \$1 (population has measure 1) and increase SWF (in \$ terms) by  $\int_i g_i$

$g_i$  typically depend on tax system (endogenous variable)

Utilitarian case:  $g_i$  decreases with  $z_i$  due to decreasing marginal utility of consumption

Rawlsian case:  $g_i$  concentrated on most disadvantaged (typically those with  $z_i = 0$ )

## OPTIMAL LINEAR TAX RATE: INDIVIDUAL PROBLEM

Disposable income (consumption):  $c = (1 - \tau) \cdot z + R$  with  $\tau$  linear tax rate and  $R$  demogrant funded by taxes  $\tau Z$  with  $Z$  aggregate earnings

Population of size one (continuum) with heterogeneous preferences  $u^i(c, z)$  [differences in earnings ability are built in utility function]

Individual  $i$  chooses  $z$  to maximize  $u^i((1 - \tau) \cdot z + R, z)$  labor supply choices  $z^i(1 - \tau, R)$  aggregate to economy wide earnings  $Z(1 - \tau) = \int_i z^i$  (are a function of the net-of-tax-rate).

Tax Revenue  $R(\tau) = \tau \cdot Z(1 - \tau)$  is inversely U-shaped with  $\tau$ :  
 $R(\tau = 0) = 0$  (no taxes) and  $R(\tau = 1) = 0$  (nobody works): called the Laffer Curve

# OPTIMAL LINEAR TAX RATES: PLAN

Let's look at:

- 1) The optimal linear tax formula (on all income  $z \in [0, \infty)$ ).
- 2) The revenue-maximizing rate (special case).
- 3) The top revenue-maximizing tax rate (nests previous case if top bracket starts at  $z = 0$ ).

## OPTIMAL LINEAR TAX RATE: FORMULA

Government chooses  $\tau$  to maximize

$$\int_i G[u^i((1 - \tau)z^i + \tau Z(1 - \tau), z^i)]$$

Govt FOC (using the envelope theorem as  $z^i$  maximizes  $u^i$ ):

$$0 = \int_i G'(u^i)u_c^i \cdot \left[ -z^i + Z - \tau \frac{dZ}{d(1 - \tau)} \right],$$

$$0 = \int_i G'(u^i)u_c^i \cdot \left[ (Z - z^i) - \frac{\tau}{1 - \tau} eZ \right],$$

First term ( $Z - z^i$ ) is mechanical redistributive effect of  $d\tau$ , second term is efficiency cost due to behavioral response of  $Z$

⇒ we obtain the following optimal linear income tax formula

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + e} \quad \text{with} \quad \bar{g} = \frac{\int g_i \cdot z_i}{Z \cdot \int g_i}, \quad g_i = G'(u^i)u_c^i$$

## OPTIMAL LINEAR TAX RATE: FORMULA

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + e} \quad \text{with} \quad \bar{g} = \frac{\int g_i \cdot z_i}{Z \cdot \int g_i}, \quad g_i = G'(u^i) u_c^i$$

$0 \leq \bar{g} < 1$  if  $g_i$  is decreasing with  $z_i$  (social marginal welfare weights fall with  $z_i$ ).

$\bar{g}$  low when (a) inequality is high, (b)  $g^i \downarrow$  sharply with  $z^i$

Formula captures the equity-efficiency trade-off **robustly** ( $\tau \downarrow \bar{g}$ ,  $\tau \downarrow e$ )

Rawlsian case:  $g_i \equiv 0$  for all  $z_i > 0$  so  $\bar{g} = 0$  and  $\tau = 1/(1 + e)$

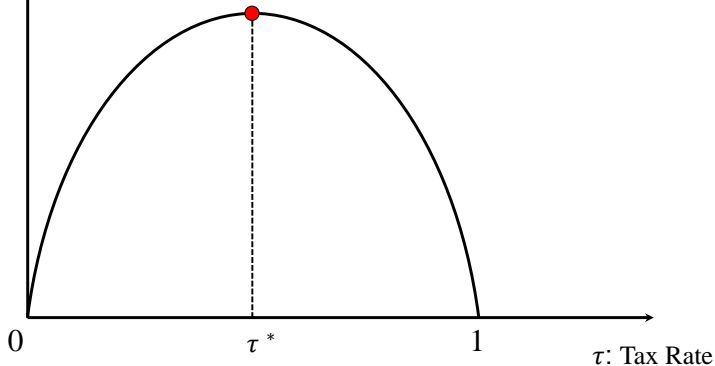
Rawlsian optimum = top of Laffer curve if  $\min_i u^i$  agent earns  $z_i = 0$ .

## Laffer Curve

Tax  
Revenue  
R

$$R = \tau \cdot Z(1 - \tau)$$

$$\tau^* = \frac{1}{1 + e} \text{ with } e = \frac{1 - \tau}{Z} \cdot \frac{dZ}{d(1 - \tau)}$$



## OPTIMAL TOP INCOME TAX RATE (SAEZ '01)

Consider constant MTR  $\tau$  above fixed  $z^*$ . Goal is to derive optimal  $\tau$

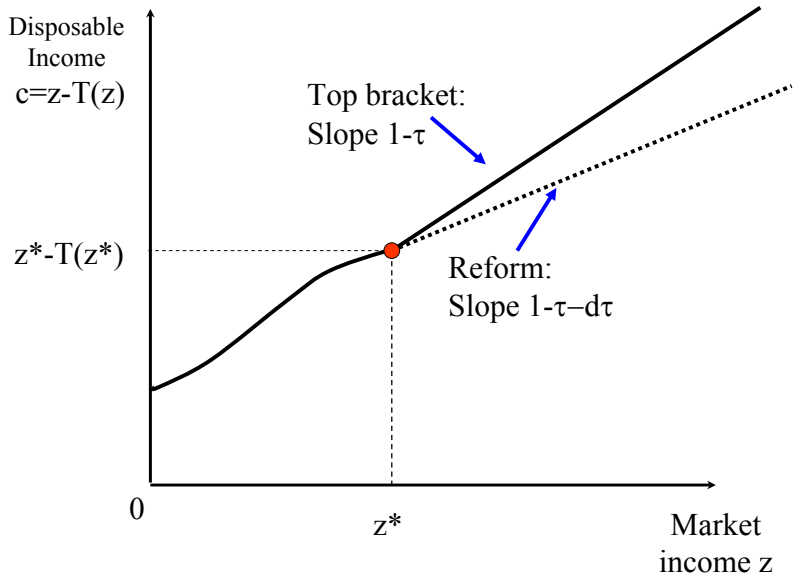
Assume w.l.o.g there is a continuum of measure one of individuals above  $z^*$

Let  $z(1 - \tau)$  be their average income [depends on net-of-tax rate  $1 - \tau$ ], with elasticity  $e = [(1 - \tau)/z] \cdot dz/d(1 - \tau)$

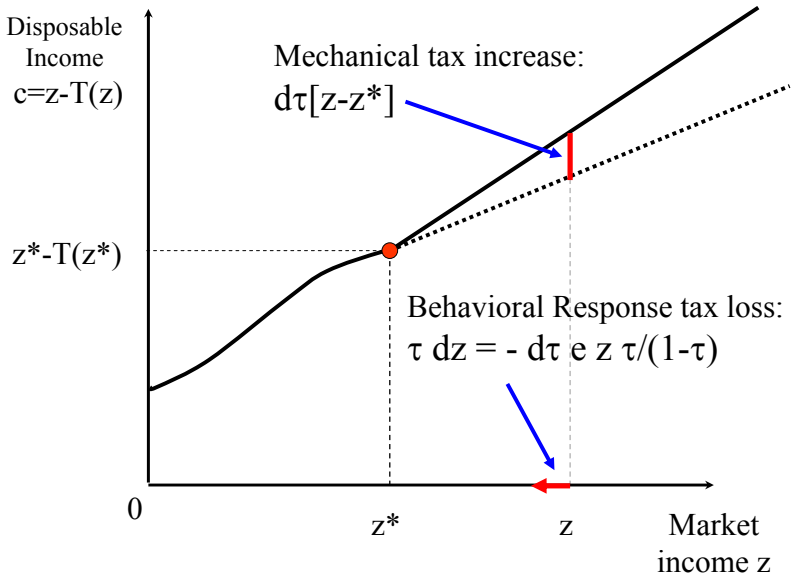
! Careful, what is  $e$ ?

Note that  $e$  is a mix of income and substitution effects (see Saez '01)

# Optimal Top Income Tax Rate (Mirrlees '71 model)



# Optimal Top Income Tax Rate (Mirrlees '71 model)



## OPTIMAL TOP INCOME TAX RATE

Consider small  $d\tau > 0$  reform above  $z^*$ .

1) **Mechanical increase** in tax revenue:

$$dM = [z - z^*]d\tau$$

2) **Welfare effect**:

$$dW = -\bar{g}dM = -\bar{g}[z - z^*]d\tau$$

where  $\bar{g}$  is the social marginal welfare weight for top earners

3) **Behavioral response** reduces tax revenue:

$$\begin{aligned} dB = \tau \cdot dz &= -\tau \frac{dz}{d(1-\tau)} d\tau = -\frac{\tau}{1-\tau} \cdot \frac{1-\tau}{z} \frac{dz}{d(1-\tau)} \cdot zd\tau \\ &\Rightarrow dB = -\frac{\tau}{1-\tau} \cdot e \cdot zd\tau \end{aligned}$$

## OPTIMAL TOP INCOME TAX RATE

$$dM + dW + dB = d\tau \left[ (1 - \bar{g})[z - z^*] - e \frac{\tau}{1 - \tau} z \right]$$

Optimal  $\tau$  such that  $dM + dW + dB = 0 \Rightarrow$

$$\frac{\tau}{1 - \tau} = \frac{(1 - \bar{g})[z - z^*]}{e \cdot z}$$

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + a \cdot e} \quad \text{with} \quad a = \frac{z}{z - z^*}$$

Optimal  $\tau \downarrow \bar{g}$  [redistributive tastes]

Optimal  $\tau \downarrow$  with  $e$  [efficiency]

Optimal  $\tau \downarrow a$  [thinness of top tail]

## OPTIMAL LINEAR RATES: RECAP

1) The optimal linear tax formula (on all income  $z \in [0, \infty)$ ):

$$\tau^* = \frac{1 - \bar{g}}{1 - \bar{g} + e} \quad \text{with} \quad \bar{g} = \frac{\int g_i \cdot z_i}{Z \cdot \int g_i}, \quad g_i = G'(u^i) u_c^i$$

2) The revenue-maximizing rate (special case if  $\bar{g} = 0$ , i.e., if  $g_i = 0$  for all  $z_i \neq 0$ ).

$$\tau^R = \frac{1}{1 + e}$$

3) The top revenue-maximizing tax rate (equal to  $\tau^*$  if  $z^* = 0$ ).

$$\tau^{top} = \frac{1 - \bar{g}}{1 - \bar{g} + a \cdot e} \quad \text{with} \quad a = \frac{z}{z - z^*}$$

## SUFFICIENT STATS FORMULA

Pause for a bit: did we say anything about underlying characteristics of people?

Note how general the formula is!

Sufficient statistics, observables only.

## ZERO TOP RATE RESULT

Suppose top earner earns  $z^T$

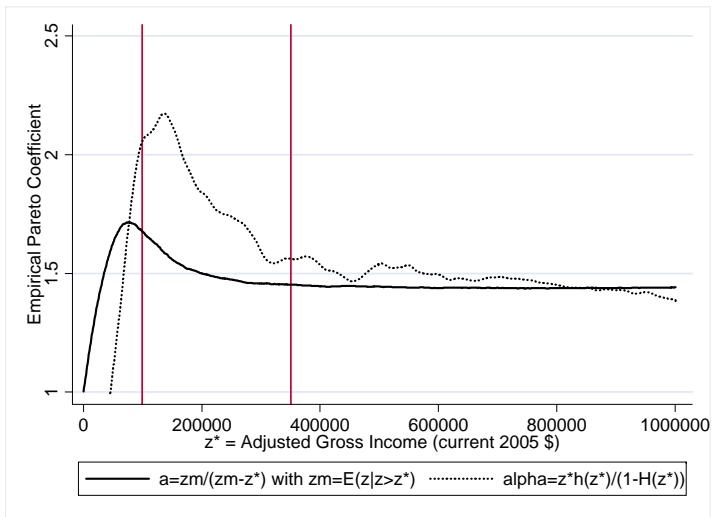
When  $z^* \rightarrow z^T \Rightarrow z \rightarrow z^T$

$$dM = d\tau[z - z^*] \ll dB = d\tau \cdot e \cdot \frac{\tau}{1 - \tau} z \quad \text{when } z^* \rightarrow z^T$$

Intuition: extra tax applies only to earnings above  $z^*$  but behavioral response applies to full  $z \Rightarrow$

Optimal  $\tau$  should be zero when  $z^*$  close to  $z^T$  (Sadka-Seade zero top rate result) but result applies only to top earner

Top is uncertain: If actual distribution is finite draw from an underlying Pareto distribution then expected revenue maximizing rate is  $1/(1 + a \cdot e)$  (Diamond and Saez JEP'11)



Source: Diamond and Saez JEP'11

## OPTIMAL TOP INCOME TAX RATE

Empirically:  $a = z/(z - z^*)$  very stable above  $z^* = \$400K$

Pareto distribution  $1 - F(z) = (k/z)^\alpha$ ,  $f(z) = \alpha \cdot k^\alpha / z^{1+\alpha}$ , with  $\alpha$  Pareto parameter

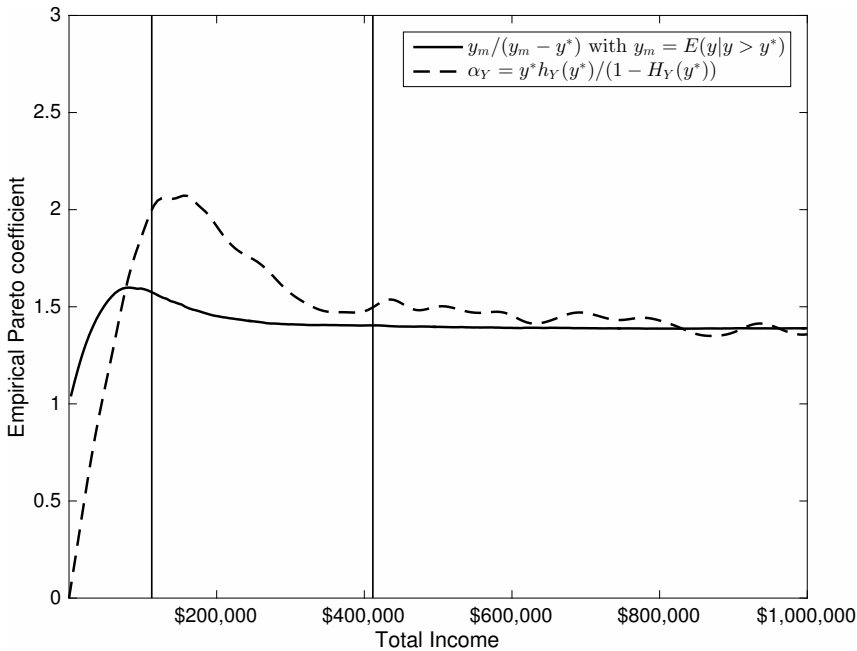
$$z(z^*) = \frac{\int_{z^*}^{\infty} sf(s) ds}{\int_{z^*}^{\infty} f(s) ds} = \frac{\int_{z^*}^{\infty} s^{-\alpha} ds}{\int_{z^*}^{\infty} s^{-\alpha-1} ds} = \frac{\alpha}{\alpha - 1} \cdot z^*$$

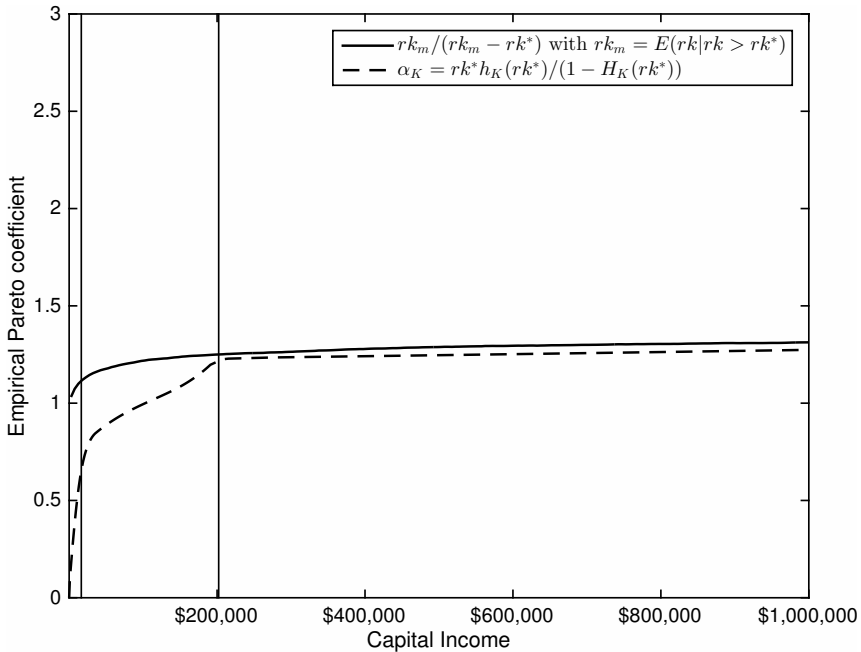
$\alpha = z/(z - z^*) = a$  measures *thinness* of top tail of the distribution

Empirically  $a \in (1.5, 3)$ , US has  $a = 1.5$ , Denmark has  $a = 3$

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + a \cdot e}$$

Only difficult parameter to estimate is  $e$





## TOP TAX REVENUE MAXIMIZING TAX RATE

Utilitarian criterion with  $u_c \rightarrow 0$  when  $c \rightarrow \infty \Rightarrow \bar{g} \rightarrow 0$  when  $z^* \rightarrow \infty$

Rawlsian criterion (maximize utility of worst off person)  $\Rightarrow \bar{g} = 0$  for any  $z^* > \min(z)$

In the end,  $\bar{g}$  reflects the value that society puts on marginal consumption of the rich

$\bar{g} = 0 \Rightarrow$  Tax Revenue Maximizing Rate  $\tau = 1/(1 + a \cdot e)$  (upper bound on top tax rate)

Example:  $a = 2$  and  $e = 0.25 \Rightarrow \tau = 2/3 = 66.7\%$

Laffer linear rate is a special case with  $z^* = 0$ ,  $z^m/z^* = \infty = a/(a - 1)$  and hence  $a = 1$ ,  $\tau = 1/(1 + e)$

## EXTENSIONS AND LIMITATIONS

- 1) Model includes only intensive earnings response. Extensive earnings responses [entrepreneurship decisions, migration decisions]  $\Rightarrow$  Formulas can be modified
  - 2) Model does not include **fiscal externalities**: part of the response to  $d\tau$  comes from **income shifting** which affects other taxes  $\Rightarrow$  Formulas can be modified
  - 3) Model does not include **classical externalities**: (a) charitable contributions, (b) positive spillovers (trickle down) [top earners underpaid], (c) negative spillovers [top earners overpaid]
- Classical general equilibrium effects on prices are NOT externalities and do not affect formulas [Diamond-Mirrlees AER '71, Saez JpubE '04]

## GENERAL NON-LINEAR INCOME TAX $T(z)$

- (1) Lumpsum grant given to everybody equal to  $-T(0)$
- (2) Marginal tax rate schedule  $T'(z)$  describing how (a) lump-sum grant is taxed away, (b) how tax liability increases with income

Let  $H(z)$  be the income CDF [population normalized to 1] and  $h(z)$  its density [endogenous to  $T(\cdot)$ ]

Let  $g(z)$  be the social marginal value of consumption for taxpayers with income  $z$  in terms of public funds [formally  $g(z) = G'(u) \cdot u_c / \lambda$ ]: no income effects  $\Rightarrow \int g(z)h(z)dz = 1$

Redistribution valued  $\Rightarrow g(z)$  decreases with  $z$

Let  $G(z)$  the *average* social marginal value of  $c$  for taxpayers with income above  $z$  [ $G(z) = \int_z^\infty g(s)h(s)ds / (1 - H(z))$ ]



## GENERAL NON-LINEAR INCOME TAX

Assume away income effects  $\varepsilon^c = \varepsilon^u = e$  [Diamond AER'98 shows this is the key theoretical simplification]

Consider small reform: increase  $T'$  by  $d\tau$  in small band  $z$  and  $z + dz$

Mechanical effect  $dM = dzd\tau[1 - H(z)]$

Welfare effect  $dW = -dzd\tau[1 - H(z)]G(z)$

Behavioral effect: substitution effect  $\delta z$  inside small band  $[z, z + dz]$ :  
$$dB = h(z)dz \cdot T' \cdot \delta z = -h(z)dz \cdot T' \cdot d\tau \cdot z \cdot e_{(z)} / (1 - T')$$

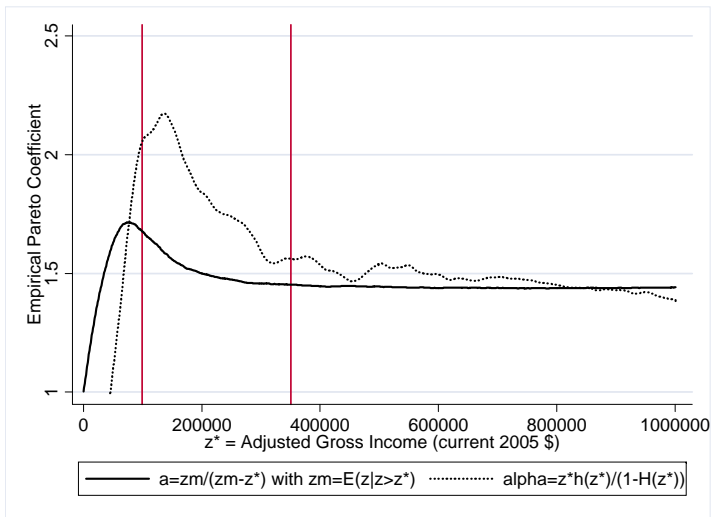
Optimum  $dM + dW + dB = 0$

## GENERAL NON-LINEAR INCOME TAX

$$T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e_{(z)}}$$

- 1)  $T'(z)$  decreases with  $e_{(z)}$  (elasticity efficiency effects)
- 2)  $T'(z)$  decreases with  $\alpha(z) = (zh(z))/(1 - H(z))$  (local Pareto parameter)
- 3)  $T'(z)$  decreases with  $G(z)$  (redistributive tastes)

Asymptotics:  $G(z) \rightarrow \bar{g}$ ,  $\alpha(z) \rightarrow a$ ,  $e_{(z)} \rightarrow e \Rightarrow$  Recover top rate formula  
 $\tau = (1 - \bar{g}) / (1 - \bar{g} + a \cdot e)$



Source: Diamond and Saez JEP'11

## Negative Marginal Tax Rates Never Optimal

Suppose  $T' < 0$  in band  $[z, z + dz]$

Increase  $T'$  by  $d\tau > 0$  in band  $[z, z + dz]$ :  $dM + dW > 0$  and  $dB > 0$   
because  $T'(z) < 0$

⇒ Desirable reform

⇒  $T'(z) < 0$  cannot be optimal

EITC schemes are not desirable in Mirrlees '71 model

## MIRRLEES MODEL

The difference to before: we need to specify the *structural primitives*.

Key simplification is the lack of income effects (Diamond, 1998). We look into income effects next time.

Individual utility:  $c - v(l)$ ,  $l$  is labor supply.

Skill  $n$  is exogenously given, equal to marginal productivity. Earnings are  $z = nl$ .

Density is  $f(n)$  and CDF  $F(n)$  on  $[0, \infty)$ .

Entry into contract theory/mechanism design here: The government does not observe **skill**. Tax is based on income  $z$ ,  $T(z)$ .

What happens if we had a tax  $T(n)$  available?

Why did we not talk about this in the earlier derivations? Did we ignore the incentive compatibility constraints?

## Elasticity of labor to taxes

Recall we derive elasticities on the linearized budget set. If marginal tax rate is  $\tau$ , labor supply is:  $l = l(n(1 - \tau))$ . Why the  $n(1 - \tau)$ ? Why *only*  $n(1 - \tau)$ ?

FOC of the agent for labor supply:

$$n(1 - \tau) = v'(l)$$

Totally differentiate this (key thing: skill is fixed!)

$$d(n(1 - \tau)) = v''(l)dl$$

$$\Rightarrow e = \frac{dl}{d(n(1 - \tau))} \frac{(1 - \tau)n}{l} = \frac{(1 - \tau)n}{lv''(l)} = \frac{v'(l)}{lv''(l)}$$

Is this compensated? uncompensated?

# Direct Revelation Mechanism and Incentive Compatibility

We want to max social welfare and have exogenous revenue requirement (non transfer-related  $E$ ).

We imagine a direct revelation mechanism. Every agent comes to government, reports a type  $n'$ . We assign allocations as a function of the report.  $c(n')$ ,  $z(n')$ ,  $u(n')$ . Why are we not assigning labor  $l(n')$ ?

What are the constraints in this problem?

Feasibility (net resources sum to zero):  $\int_n c_n f(n) dn \geq n l_n f(n) dn - E$ .

Incentive compatibility:

## Direct Revelation Mechanism and Incentive Compatibility

We want to max social welfare and have exogenous revenue requirement (non transfer-related  $E$ ).

We imagine a direct revelation mechanism. Every agent comes to government, reports a type  $n'$ . We assign allocations as a function of the report.  $c(n')$ ,  $z(n')$ ,  $u(n')$ . Why are we not assigning labor  $l(n')$ ?

What are the constraints in this problem?

Feasibility (net resources sum to zero):  $\int_n c_n f(n) dn \geq n l_n f(n) dn - E$ .

Incentive compatibility:

$$c(n) - v \left( \frac{z(n)}{n} \right) \geq c(n') - v \left( \frac{z(n')}{n} \right) \quad \forall n, n'$$

That's a lot of constraints!

## Envelope Theorem and First order Approach

Replace the infinity of constraints with agents' first-order condition. If we take derivative of utility wrt type  $n$  at *truth-telling*

$$\frac{du_n}{dn} = \left( c'(n) - \frac{z'(n)}{n} v' \left( \frac{z(n)}{n} \right) \right) \frac{dn'}{dn} + \frac{z(n)}{n^2} v' \left( \frac{z(n)}{n} \right)$$

What if report is optimally chosen?

Envelope condition:

$$\frac{du_n}{dn} = \frac{l_n v'(l_n)}{n}$$

Will replace infinity of constraints.

Is necessary, but what about sufficiency?

## Full Optimization Program

$$\max_{c_n, u_n, z_n} \int_n G(u_n) f(n) dn \quad \text{s.t.} \quad \int_n c_n f(n) dn \leq \int_n n l_n f(n) dn - E$$

$$\text{and s.t.} \quad \frac{du_n}{dn} = \frac{l_n v'(l_n)}{n}$$

State variable:  $u_n$ .

Control variables:  $l_n$ , with  $c_n = u_n + v(l_n)$ .

Why am I suddenly saying  $l_n$  is a control?

Use optimal control.

# Hamiltonian and Optimal Control

The Hamiltonian is:

$$H = [G(u_n) + p \cdot (nl_n - u_n - v(l_n))]f(n) + \phi(n) \cdot \frac{l_n v'(l_n)}{n}$$

$p$ : multiplier on the resource constraint.

$\phi(n)$ : multiplier on the envelope condition ("costate"). Depends on  $n$ !

FOCs:

$$\frac{\partial H}{\partial l_n} = p \cdot [n - v'(l_n)]f(n) + \frac{\phi(n)}{n} \cdot [v'(l_n) + l_n v''(l_n)] = 0$$

$$\frac{\partial H}{\partial u_n} = [G'(u_n) - p]f(n) = -\frac{d\phi(n)}{dn}$$

Transversality:  $\lim_{n \rightarrow \infty} \phi(n) = 0$  and  $\phi(0) = 0$ .

## Rearranging the FOCs

Take the integral of the FOC wrt  $u_n$  to solve for  $\phi(n)$ :

$$-\phi(n) = \int_n^{\infty} [p - G'(u_m)] f(m) dm$$

Integrate this same FOC over the full space, using transversality conditions:

$$p = \int_0^{\infty} G'(u_n) f(m) dm$$

What does this say?

How can we make the tax rate appear? Use the agent's FOC.

$$n - v'(l_n) = nT'(z_n)$$

## Obtaining the Optimal Tax Formula

Recall that  $e = \frac{(1 - T'(z_n))n}{l v''(l)}$

Rearranging the last term in the FOC for  $l_n$ :

$$[v'(l_n) + l_n v''(l_n)]/n = [1 - T'(z_n)][1 + 1/e]$$

Let  $g_m \equiv G'(u_m)/p$  be the marginal social welfare weight on type  $m$ .

Then, the FOC for  $l_n$  becomes:

$$\frac{T'(z_n)}{1 - T'(z_n)} = \left(1 + \frac{1}{e}\right) \cdot \left(\frac{\int_n^\infty (1 - g_m) dF(m)}{nf(n)}\right)$$

This is the Diamond (1998) formula.

What is different from the previously derived formula *à la* Saez (2001)?

## Let's go from types to observable income

How do we go from type distribution to income distribution?

Under linearized tax schedule, earnings are a function  $z_n = nl/(n(1 - \tau))$ .

How do earnings vary with type?

$$\frac{dz_n}{dn} = l + (1 - \tau)n \frac{dl}{d(m(1 - \tau))} = l_n \cdot (1 + e)$$

(intuition?)

Let  $h(z)$  be the density of earnings, with CDF  $H(z)$ . The following relation must hold:

$$h(z_n)dz_n = f(n)dn$$

$$f(n) = h(z_n)l_n(1 + e) \Rightarrow ng(n) = z_nh(z_n)(1 + e)$$

Let's substitute income distributions for type distributions in the formula.

## Optimal Tax Formula with No Income Effects

$$\begin{aligned}\frac{T'(z_n)}{1 - T'(z_n)} &= \left(1 + \frac{1}{e_{(z_n)}}\right) \left(\frac{\int_n^\infty (1 - g_m) dF(m)}{nf(n)}\right) \quad (\text{primitives}) \\ &= \frac{1}{e_{(z_n)}} \left(\frac{1 - H(z_n)}{z_n h(z_n)}\right) \cdot (1 - G(z_n)) \quad (\text{incomes})\end{aligned}$$

where:

$$G(z_n) = \frac{\int_n^\infty g_m dF(m)}{1 - F(n)} = \frac{\int_{z_n}^\infty g_m dH(z_m)}{1 - H(z_n)}$$

is the average marginal social welfare weight on individuals with income above  $z_n$  (change of variables to income distributions in last equality).

Rearrange, use definition of Pareto parameter  $\alpha(z) = (zh(z))/(1 - H(z))$  to get same formula as before:

$$T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e_{(z)}}$$

## Recap:

“Mechanism design approach” requires you to specify *primitives* (utility function, uni-dimensional heterogeneity) as done in Mirrlees (1971).

“Sufficient stats approach” captures arbitrary heterogeneity conditional on  $z$  as long as well-behaved elasticities.

Yield same formula if can make the link between types and income distributions.

$$\begin{aligned}\frac{T'(z_n)}{1 - T'(z_n)} &= \left(1 + \frac{1}{e_{(z_n)}}\right) \left(\frac{\int_n^\infty (1 - g_m) dF(m)}{nf(n)}\right) \quad (\text{primitives}) \\ &= \frac{1}{e_{(z_n)}} \left(\frac{1 - H(z_n)}{z_n h(z_n)}\right) \cdot (1 - G(z_n)) \quad (\text{incomes})\end{aligned}$$

## NUMERICAL SIMULATIONS

$H(z)$  [and also  $G(z)$ ] endogenous to  $T(\cdot)$ . Calibration method (Saez Restud '01):

Specify utility function (e.g. constant elasticity):

$$u(c, z) = c - \frac{1}{1 + \frac{1}{e}} \cdot \left(\frac{z}{n}\right)^{1 + \frac{1}{e}}$$

Individual FOC  $\Rightarrow z = n^{1+e}(1 - T')^e$

Calibrate the exogenous skill distribution  $F(n)$  so that, using **actual**  $T'(\cdot)$ , you recover **empirical**  $H(z)$

Use Mirrlees '71 tax formula (expressed in terms of  $F(n)$ ) to obtain the optimal tax rate schedule  $T'$ .

## NUMERICAL SIMULATIONS

$$\frac{T'(z(n))}{1 - T'(z(n))} = \left(1 + \frac{1}{e}\right) \left(\frac{1}{nf(n)}\right) \int_n^\infty \left[1 - \frac{G'(u(m))}{\lambda}\right] f(m) dm,$$

Iterative Fixed Point method: start with  $T'_0$ , compute  $z^0(n)$  using individual FOC, get  $T^0(0)$  using govt budget, compute  $u^0(n)$ , get  $\lambda$  using  $\lambda = \int G'(u)f$ , use formula to estimate  $T'_1$ , iterate till convergence

Fast and effective method (Brewer-Saez-Shepard '10)

## NUMERICAL SIMULATION RESULTS

$$T'(z) = \frac{1 - G(z)}{1 - G(z) + \alpha(z) \cdot e(z)}$$

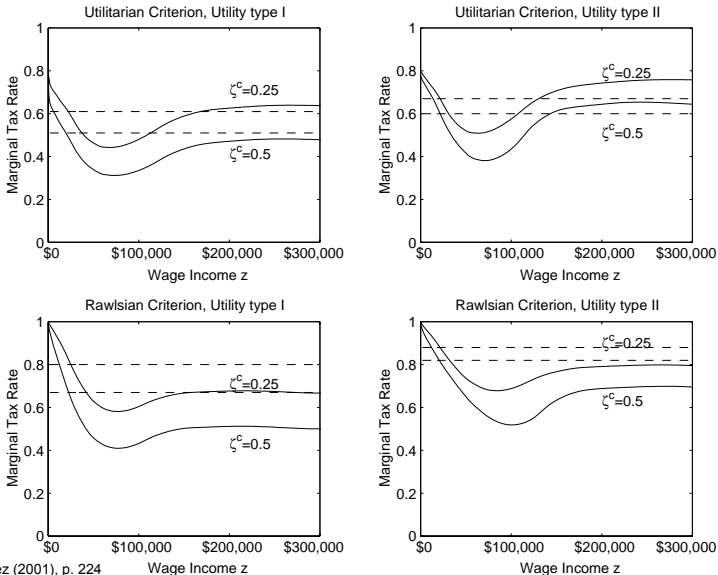
Take utility function with  $e$  constant

2)  $\alpha(z) = (zh(z))/(1 - H(z))$  is inversely U-shaped empirically

3)  $1 - G(z)$  increases with  $z$  from 0 to 1 ( $\bar{g} = 0$ )

⇒ Numerical optimal  $T'(z)$  is U-shaped with  $z$ : reverse of the general results  $T' = 0$  at top and bottom [Diamond AER'98 gives theoretical conditions to get U-shape]

FIGURE 5 – Optimal Tax Simulations



Source: Saez (2001), p. 224

## EXTENSION 1: MIGRATION EFFECTS

Tax rates may affect migration (evidence on this next time).

Migration issues may be particularly important at the top end (brain drain).

Some theory papers (Mirrlees '82, Lehmann-Simula QJE'14). Here:  
Simplified Mirrlees (1982) model.

Earnings  $z$  are fixed, conditional on residence.

$P(c|z)$  is number of residents earning  $z$  when disposable income is  $c$ , with  
 $c = z - T(z)$ .

Consider small tax reform  $dT(z)$  for those earning  $z$ .

What is migration responding to? Marginal taxes?

## ELASTICITY OF MIGRATION TO TAXES

Mechanical effect net of welfare is:  $M + W = (1 - g(z))P(c|z)dT$ .

Why? Where is utility effect of changing country induced by taxes?

Migration responds to average taxes (or total taxes, since income fixed).

$$\eta_m(z) = \frac{\partial P(c|z)}{\partial c} \frac{z - T(z)}{P(c|z)}$$

Fiscal cost of raising taxes by  $dT(z)$  is:  $B = -\frac{T(z)}{z - T(z)} \cdot P(c|z) \cdot \eta_m$

Optimal tax is where  $M + W + B = 0$ :

$$\frac{T(z)}{z - T(z)} = \frac{1}{\eta_m(z)} \cdot (1 - g(z))$$

What determines the elasticity  $\eta_m(z)$ ?

## MIGRATION EFFECTS IN THE STANDARD MODEL

$\eta_m(z)$  depends on size of jurisdiction: large for cities, zero worldwide  $\Rightarrow$   
(1) Redistribution easier in large jurisdictions, (2) Tax coordination across countries increases ability to redistribute (big issue currently in EU), (3) visa system, cost of migration, ...

Top revenue maximizing tax rate formula (Brewer-Saez-Shepard '10):

$$\tau = \frac{1}{1 + a \cdot e + \bar{\eta}^m}$$

where  $\bar{\eta}^m$  is the elasticity of top earners to disposable income.

## EXTENSION 2: RENT SEEKING EFFECTS

Pay may not be equal to the marginal economic product for top income earners. Why? Overpaid or underpaid?

Piketty, Saez, and Stantcheva (2014) "A Tale of Three Elasticities."

Actual output is  $y$ , but individual only receives share  $\eta$  of actual output. To increase either productive effort or rent-seeking, effort is required.

$$u^i(c, \eta, y) = c - h_i(y) - k_i(\eta)$$

Define bargained earnings:  $b = (\eta - 1)y$ .

Average bargaining is  $E(b)$ , extracted equally from everyone else (good assumption?) Means  $E(b)$  can be perfectly canceled by  $-T(0)$ .

## RENT SEEKING ELASTICITIES

Given tax, individual maximizes:

$$u^i(c, y, \eta) = \eta \cdot y - T(\eta \cdot y) - h_i(y) - k_i(\eta)$$

What will  $y_i$  and  $\eta_i$  depend on?

Average reported income, productive income and bargained earnings in the top bracket:

$$z(1 - \tau), \quad y(1 - \tau), \quad \eta(1 - \tau)$$

Total compensation elasticity  $e$ :  $e = \frac{1-\tau}{z} \frac{dz}{d(1-\tau)}$  (what is it driven by?)

Real labor supply elasticity  $e_y$ :  $e_y = \frac{1-\tau}{y} \frac{dy}{d(1-\tau)} \geq 0$ .

Thus the bargaining elasticity component  $e_b = \frac{db}{d(1-\tau)} \frac{1-\tau}{z} = s \cdot e$  with

$$s = \frac{db/d(1-\tau)}{dz/d(1-\tau)}$$

$s$  and  $e_b$  positive if  $\eta > 1$ .

## OPTIMAL TAX RATE WITH RENT SEEKING

Suppose rent-seeking only at the top,  $E(b) = qb(1 - \tau)$  where  $q$  fraction of top earners.

Government maximizes tax revenues from top bracket earners:

$$T = \tau[y(1 - \tau) + b(1 - \tau) - z^*]q - E(b)$$

Why does  $E(b)$  enter?

$$\tau^* = \frac{1 + a \cdot e_b}{1 + a \cdot e} = 1 - \frac{a(y/z)e_y}{1 + a \cdot e}$$

How does  $\tau^*$  change with  $e$ ,  $e_y$ , and  $e_b$ ? When is  $\tau^* = 1$  optimal?

Trickle up vs trickle down: what happens to  $\tau^*$  when top earners are overpaid? Underpaid?

How would you measure  $e_b$  (even  $b$  itself?)

# OPTIMAL NON-LINEAR TAX WITH INCOME EFFECTS

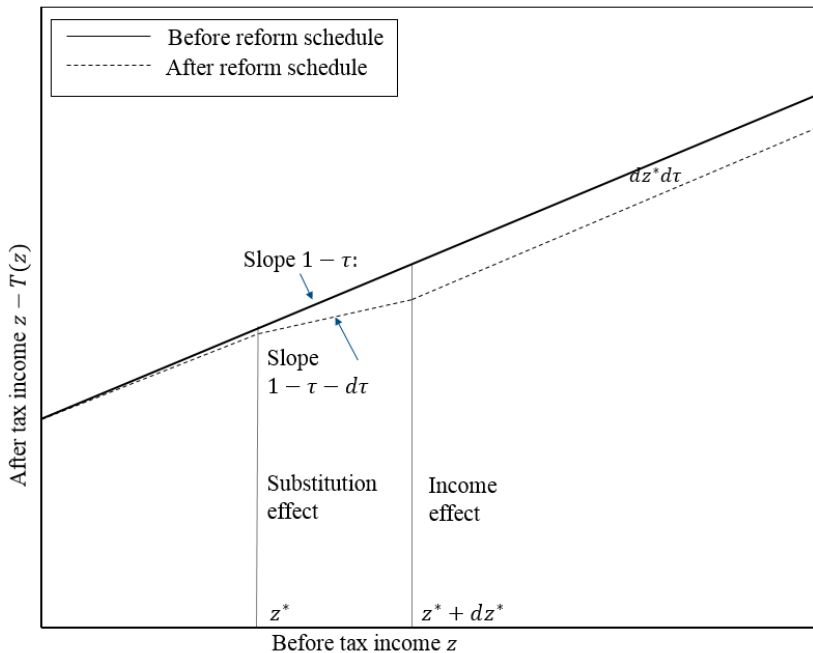
Consider effect of small reform where marginal tax rates increased by  $d\tau$  in  $[z^*, z^* + dz^*]$ .

What are the effects on tax receipts?

Mechanical effect net of welfare loss,  $M$ :

Every tax payer with income  $z$  above  $z^*$  pays additional  $d\tau dz^*$ , valued at  $(1 - g(z))d\tau dz^*$ .

$$M = d\tau dz^* \int_{z^*}^{\infty} (1 - g(z))h(z)dz$$



## BEHAVIORAL EFFECT PART 1: SUBSTITUTION

In  $[z^*, z^* + dz^*]$ , income changes by  $dz$ .

Marginal tax rate changes *directly* by  $d\tau$ , but also additionally *indirectly* by  $dT'(z) = T''(z)dz$ . Why? When is this not the case?

$$dz = -\varepsilon_{(z)}^c z^* \frac{d\tau + dT'(z)}{1 - T'(z)} \Rightarrow dz = -\varepsilon_{(z)}^c z^* \frac{d\tau}{1 - T'(z) + \varepsilon_{(z)}^c z^* T''(z)}$$

Define the *virtual density*: density that would occur at  $z$  if tax schedule replaced by linearized tax schedule. What is the linearized schedule  $(\tau, R)$  such that income is  $(1 - \tau)z + R$ ?

$$\frac{h^*(z)}{1 - T'(z)} = \frac{h(z)}{1 - T'(z) + \varepsilon_{(z)}^c z^* T''(z)}$$

# BEHAVIORAL EFFECT PART 1: SUBSTITUTION

Overall elasticity/substitution effect is then:

$$E = -\varepsilon_{(z)}^c z^* \frac{T'(z)}{1 - T'(z)} h^*(z^*) d\tau dz^*$$

Can derive expression without taking into account endogenous (indirect) change in marginal tax rates if use the virtual density instead of true one.

## BEHAVIORAL EFFECT PART 2: INCOME EFFECT

Taxpayers with income above  $z^*$  pay  $-dR = d\tau dz^*$  additional taxes. Their change in income is:

$$dz = -\varepsilon_{(z)}^c z \frac{T'' dz}{1 - T'} - \eta \frac{d\tau dz^*}{1 - T'(z)} \Rightarrow dz = -\eta \frac{d\tau dz^*}{1 - T'(z) + z\varepsilon_{(z)}^c T''(z)}$$

Why?

Total income effect response:

$$I = d\tau dz^* \int_{z^*}^{\infty} -\eta_{(z)} \frac{T'(z)}{1 - T'(z)} h^*(z) dz$$

At the optimum:  $M + E + I = 0$ .

## PUTTING THE EFFECTS TOGETHER

$$\frac{T'(z)}{1 - T'(z)} = \frac{1}{\varepsilon_{(z)}^c} \left( \frac{1 - H(z^*)}{z^* h^*(z^*)} \right)$$
$$\times \left[ \int_{z^*}^{\infty} (1 - g(z)) \frac{h(z)}{1 - H(z^*)} dz + \int_{z^*}^{\infty} -\eta \frac{T'(z)}{1 - T'(z)} \frac{h^*(z)}{1 - H(z^*)} dz \right]$$

First-order differential equation. See Saez (2001) Appendix for solution (is standard).

Change of variable from  $z$  to  $n$ ?

Recall with a linear tax:  $\frac{\dot{z}_n}{z_n} = \frac{1 + \varepsilon_{(z_n)}^u}{n}$ .

What happens with nonlinear tax? See Saez (2001) Appendix for derivation.

$$\frac{\dot{z}_n}{z_n} = \frac{1 + \varepsilon_{(z_n)}^u}{n} - \dot{z}_n \frac{T''(z_n)}{1 - T'(z_n)} \varepsilon_{z(n)}^c$$

## ATKINSON-STIGLITZ THEOREM

Famous Atkinson-Stiglitz JpubE' 76 shows that

$$\max_{t, T(\cdot)} SWF = \max_{t=0, T(\cdot)} SWF$$

(i.e, commodity taxes not useful) under two assumptions on utility functions  $u^h(c_1, \dots, c_K, z)$

- 1) Weak separability between  $(c_1, \dots, c_K)$  and  $z$  in utility
- 2) Homogeneity across individuals in the sub-utility of consumption  $v(c_1, \dots, c_K)$  [does not vary with  $h$ ]

$$(1) \text{ and } (2): \quad u^h(c_1, \dots, c_K, z) = U^h(v(c_1, \dots, c_K), z)$$

Original proof was based on optimum conditions, new straightforward proof by Laroque EL '05, and Kaplow JpubE '06.

## ATKINSON-STIGLITZ THEOREM PROOF

Let  $V(y, p + t) = \max_c v(c_1, \dots, c_K)$  st  $(p + t) \cdot c \leq y$  be the indirect utility of consumption  $c$  [common to all individuals]

Start with  $(T(\cdot), t)$ . Let  $c(t)$  be consumer choice.

Replace  $(T(\cdot), t)$  with  $(\bar{T}(\cdot), t = 0)$  where  $\bar{T}(z)$  such that  $V(z - T(z), p + t) = V(z - \bar{T}(z), p) \Rightarrow$  Utility  $U^h(V, z)$  and labor supply choices  $z$  unchanged for all individuals.

Attaining  $V(z - \bar{T}(z), p)$  at price  $p$  costs at least  $z - \bar{T}(z)$

Consumer also attains  $V(z - \bar{T}(z), p) = V(z - T(z), p + t)$  when choosing  $c(t) \Rightarrow z - \bar{T}(z) \leq p \cdot c(t) = z - T(z) - t \cdot c(t)$

$\Rightarrow \bar{T}(z) \geq T(z) + t \cdot c(t)$ : the government collects more taxes with  $(\bar{T}(\cdot), t = 0)$

## ATKINSON-STIGLITZ INTUITION

With separability and homogeneity, conditional on earnings  $z$ , consumption choices  $c = (c_1, \dots, c_K)$  do not provide any information on ability

⇒ Differentiated commodity taxes  $t_1, \dots, t_K$  create a tax distortion with no benefit ⇒ Better to do all the redistribution with the individual income tax

Note: With weaker linear income taxation tool (Diamond–Mirrlees AER '71, Diamond JpubE '75), need  $v(c_1, \dots, c_K)$  homothetic (linear Engel curves, Deaton EMA '81) to obtain no commodity tax result

[Unless Engel curves are linear, commodity taxation can be useful to “non-linearize” the tax system]

## Generalization of Atkinson-Stiglitz to Heterogeneous Tastes – Saez (2002)

Can we generalize AS to case with heterogeneous consumption preferences?

Individuals indexed by  $h$ , utility  $U(c, z)$  with  $c = (c_1, \dots, c_K)$ .

Nonlinear income tax  $T(z)$ .

Pre-tax prices:  $p$ , post-tax price:  $q = p + t$ .

Budget constraint  $q \cdot c \leq z - T(z)$ .

Demands:  $c^h(q, R, z)$ , labor supply  $z^h(q, T)$ , indirect utility  $v^h(q, R, z)$ .

Suppose that  $T(z)$  is optimally chosen at zero commodity taxation  $p = q$  to max

$$W = \sum_h \alpha^h v^h(p, z^h - T(z^h), z^h) \quad \text{s.t.} \quad \sum_h T(z^h) \geq E$$

Marginal social welfare weight  $g^h = \alpha^h v_R^h / \lambda$ .

## Can commodity taxation improve welfare?

Imagine  $dt_1$ .

Mechanical revenue effect:  $dM_1 = \sum_h c_1^h dt_1 = C_1 dt_1$ .

Welfare effect (envelope theorem):  $dU_1 = - \sum_h g^h c_1^h dt_1$ .

Behavioral labor supply response:  $dB_1 = - \sum_h T'(z^h) dz_{t_1}^h$  with  $dz_{t_1}^h = dt_1 \frac{\partial z^h}{\partial q_1}$ .

Why no behavioral response on revenue from changes in consumption?

If no commodity tax introduction can increase welfare, need

$$dW = dM_1 + dU_1 + dB_1 = 0.$$

## Can commodity taxation improve welfare? (II)

Find a small income tax reform that “mimics” commodity tax change:

$$dT(z) = C_1(z)dt_1.$$

This reform has zero first order welfare impact, why?

Mechanical Revenue effect:

$$dM_T = \sum_h dT(z^h) = \sum_h C_1(z^h)dt_1 = C_1 dt_1 = dM_1 \text{ (why?)}$$

$$\text{Welfare effect: } dU_T = - \sum_h g^h C_1(z^h)dt_1$$

$$\text{Behavioral effect: } dB_t = - \sum_h T'(z^h)dz_T^h.$$

Subtract one from other (using that  $dW_T = 0$ ).

## Can commodity taxation improve welfare? (III)

$$\frac{dW}{dt_1} = \underbrace{-\sum_h g^h [c_1^h - C_1(z^h)]}_{\text{Pure Welfare Effect}} + \underbrace{\sum_h T'(z^h) \left[ \frac{dz_T^h}{dT(z^h)} \cdot \frac{dT(z^h)}{dt_1} - \frac{dz_1^h}{dt_1} \right]}_{\text{Behavioral Effect}}$$

- Pure welfare effect is zero if conditional on  $z$ ,  $g^h$  and  $c^h$  are uncorrelated.
- What does this mean? Is this reasonable? (recall weights are "generalized" since  $\alpha^h$  depends on  $h$  directly).
- Young/old? medical expenses?
- Always satisfied in "standard AS" assumptions.

## Behavioral Effects under Commodity and Income Taxation

Since  $T'(z^h) \geq 0$  (remember), increasing  $dt_1 > 0$  more efficient than equivalent income tax increase if labor supply increase from commodity tax change larger than that of income tax change.

When is this the case? Can show that:

$$E[dz_{t_1}^h] = -dt_1 \left( \mathbb{E} \left[ \frac{z_c^h}{1 + T''(z)z_c^h} \frac{dc_1^h}{dz} \right] + \mathbb{E} \left[ \frac{z_R^h}{1 + T''(z)z_c^h} c_1^h \right] \right)$$

$$E[dz_T^h] = -dt_1 \left( \mathbb{E} \left[ \frac{z_c^h}{1 + T''(z)z_c^h} \right] \frac{dC_1(z)}{dz} + \mathbb{E} \left[ \frac{z_R^h}{1 + T''(z)z_c^h} \right] C_1(z) \right)$$

Need  $E[dz_{t_1}^h] = E[dz_T^h]$  for no commodity taxation.

## Assumptions needed for behavioral effects to be the same under Commodity and Income Taxation

**Assumption 2:** Conditional on  $z$ , behavioral responses  $z_c^h$  and  $z_R^h$  independent of consumption patterns  $c_1^h$  and  $\frac{dc_1^h}{dz}$ .

Do you think this holds?

**Assumption 3:** For any income level,  $E\left(\frac{dc_1^h}{dz} \mid z^h = z\right) = \frac{dC_1(z)}{dz}$ .

This is the *key* assumption. What does it say? Why is this not mechanically true?

$\frac{dC_1(z)}{dz} = \lim_{dz \rightarrow 0} \frac{E(c_1^h \mid z^h = z + dz) - E(c_1^h \mid z^h = z)}{dz}$  is cross-sectional variation in consumption of good 1 when income changes.

What is  $E\left(\frac{dc_1^h}{dz} \mid z^h = z\right)$ ?

## Assumptions needed for behavioral effects to be the same under Commodity and Income Taxation

Imagine 2 groups:

Group A:  $z^h = z$ . consume  $C_1(z)$  on average.

Group B:  $z^h = z - dz$ . Consumes on average  $dc_1 = \frac{dC_1(z)}{dz}$  less of good 1.

Group A': Individuals from group A who are *forced* to reduce their income to  $z - dz$ . Reduce their consumption relative to group A by

$$dc'_1 = E \left( \frac{dc_1^h}{dz} \mid z^h = z \right) dz.$$

Assumption 3 says: Group B = Group A' for consumption of good 1.

Always true in AS since consumption only depends on after tax income with separability  $C_1(z) = c_1^h(z)$ .

Why would Group A' not have same consumption of good 1 as Group B?

## WHEN ATKINSON-STIGLITZ ASSUMPTIONS FAIL

Thought experiment we just did was: force high earners to work less and earn only as much as low earners: if high earners consume more of good  $k$  than low earners, taxing good  $1$  is desirable.

1) High earners are “different” (since if left to choose, choose to work more. If they have a relatively higher/lower taste for good  $1$  (independently of income), tax more/less good  $1$ . [indirect tagging] Cigarettes? Fancy wine? How would you see this empirically?

2) High earners now have more leisure. If Good  $1$  positively related to leisure (consumption of  $1$  increases when leisure increases keeping after-tax income constant), tax it! [tax on holiday trips, subsidy on work related expenses such as child care]

In general Atkinson-Stiglitz assumption is a good starting place for most goods  $\Rightarrow$  Zero-rating on some goods under VAT for redistribution is inefficient and administratively burdensome [Mirrlees review]

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