Lecture 10: New Dynamic Public Finance (NDPF)

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GOALS OF THIS LECTURE

(1) New Dynamic Public Finance: A toolbox.

(2) Understand capital taxation from the point of view of NDPF

(3) Application to human capital and how to finance investments in education

SET UP: UNCERTAINTY ON EARNINGS

- So far: representative agents, ex ante heterogeneity, aggregate uncertainty
- We now consider idiosyncratic uncertainty that is not only ex ante, but unfolds over time
- Skill shocks or preference shocks
- Start with finite horizon: $t = 0, 1$
- Preferences $U(c_0, c_1(s), y(s)/s)$
- interpretation: skill shock s realized in period 1. Consumption decision c_0 in period 0 is made before the shock is realized

SET UP: FAILURE OF A-S and RESOURCE CONSTRAINT

- Note difference to time-0 shock (ex ante heterogeneity) as considered so far. Preferences would be $U(c_0(s), c_1(s), y(s)/s)$
- Under separability + homogeneity, the Atkinson-Stiglitz (1976) theorem would rule out the optimality of a capital tax.
- With the period-1 shock, we will find a downward distortion of saving to be optimal (positive capital tax).
- Technology: linear storage with rate of return $R^* = 1/q$, so that the aggregate resource constraint is

$$c_0 + q \sum_s c_1(s)p(s) \leq q \sum_s y(s)p(s) \tag{1}$$

FIRST BEST

$$\max_{c_0, c_1(s), y(s)} \sum_s U(c_0, c_1(s), y(s)/s) p(s)$$

s.t. (1)

- FOCs for $[c_0]$

$$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = \lambda$$

and for $[c_1(s)]$

$$U_{c_1(s)}(c_0, c_1(s), y(s)/s) = \lambda q$$

FIRST BEST – FULL INSURANCE

- Hence,

$$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = R^* U_{c_1(s)}(c_0, c_1(s), y(s)/s) \quad \forall s$$

$\Rightarrow$ Full Insurance

- Taking expectations on both sides

$$\mathbb{E}[U_{c_0}, c_1(s), y(s)/s] = R^* \mathbb{E}[U_{c_1(s)}(c_0, c_1(s), y(s)/s)] \tag{2}$$

- For instance, if $U(c_0, c_1(s), y(s)/s) = u(c_0) + \beta u(c_1(s)) - h(y(s)/s)$, then we obtain the usual Euler equation

$$u'(c_0) = \beta R^* \mathbb{E}[u'(c_1(s))]$$

and $c_1(s) = c_1$ for all s.

FREE SAVING GIVEN INCOME TAX

- Free saving with non-linear income tax $T(Y)$:

$$\max_{c_0, c_1(s), y(s)} \sum_s U(c_0, c_1(s), y(s)/s)p(s)$$

s.t.

$$c_0 + k_1 \leq e$$

$$c_1(s) \leq y(s) - T(y(s)) + Rk_1 \quad \forall s$$

- FOCs and $R = R^*$ yields the Euler equation

$$u'(c_0) = \beta R^* \mathbb{E}[u'(c_1(s))]$$

- If agents can freely decide how much to save in a risk-free asset with return $R = R^*$, we obtain the Euler equation as in the first best

PRIVATE INFORMATION AND INCENTIVE CONSTRAINTS

- Suppose s is private information and agents make reports $r = \sigma(s)$, where σ denotes the reporting strategy
- Truth-telling: $\sigma^*(s) = s \quad \forall s$
- Denote

$$c_1^\sigma(s) = c_1(\sigma(s))$$

  and

$$y^\sigma(s) = y(\sigma(s))$$

- The set of incentive constraints is

$$\mathbb{E}[U(c_0, c_1(s), y(s)/s)] \geq \mathbb{E}[U(c_0, c_1^\sigma(s), y^\sigma(s)/s] \quad \forall \sigma, s$$

- This is equivalent to

$$U(c_0, c_1(s), y(s)/s) \geq U(c_0, c_1(r), y(r)/s) \quad \forall r, s \tag{3}$$

SECOND BEST DYNAMIC MIRRLEES PROBLEM

- The second best (dynamic Mirrlees) problem is

$$\max_{c_0, c_1(s), y(s)/s} \mathbb{E}[U(c_0, c_1(s), y(s)/s)] \tag{4}$$

s.t.

$$c_0 + q \sum_s c_1(s) p(s) \leq q \sum_s y(s) p(s) \quad \text{(RC)}$$

and

$$U(c_0, c_1(s), y(s)/s) \geq U(c_0, c_1(r), y(r)/s) \quad \forall r, s \quad \text{(IC)}$$

FEASIBLE VARIATIONS

- (RC) and (IC) define the set F of *feasible* allocations, i.e.

$$F \equiv \{(c_0, c_1(s), y(s)) \mid (c_0, c_1(s), y(s)) \quad \text{satisfies (RC) and (IC)}$$

- Key question: Is free saving feasible? Formally, if $(c_0, c_1(s), y(s)) \in F$, does this imply that $(c_0 - \triangle, c_1(s) + R^* \triangle, y(s)) \in F$ as well, for some $\triangle \in \mathbb{R}$?
- In other words, if the agent saves a little in period 0 ($\triangle$) is she stil willing to supply the same output (i.e. not lie about s)?
- Depends on income effects in general
- For instance, suppose

$$U(c_0, c_1(s), y(s)/s) = \hat{U}(c_0, c_1(s) - h(y(s)/s))$$

Then, given c_0, just maximize $c_1(s) - h(y(s)/s)$. There are no income effects due to quasilinearity, and the above variation is feasible.

FEASIBLE VARIATIONS

- Easier to see using (IC):

$$\hat{U}(c_0 - \triangle, c_1(s) + R^*\triangle - h(y(s)/s)) \geq$$

$$\hat{U}(c_0 - \triangle, c_1(r) + R^*\triangle - h(y(r)/s))$$

  if and only if

$$c_1(s) + R^*\triangle - h(y(s)/s) \geq c_1(r) + R^*\triangle - h(y(r)/s)$$

  which is implied by the original allocation being feasible, i.e. $(c_0, c_1(s), y(s)) \in F$
- But in general, saving in period 0 has a negative income effect on labor supply in period 1 (if leisure is a normal good)
- e.g. consider preferences

$$U(c_0, c_1(s), y(s)/s) = u(c_0) + \beta u(c_1(s)) - h(y(s)/s)$$

  Additive separability + concavity of $u(.)$ mean leisure is normal (output is "inferior") and so the variation above is no longer feasible.

CAN WE FIND A FEASIBLE VARIATION?

- Free saving is not feasible with these preferences due to negative income effect on labor supply
- Consider

$$(c_0 - \triangle, c_1(s) + \delta(\triangle, s), y(s)) \tag{5}$$

with $\delta(\triangle, s)$ chosen such that (IC) is satisfied:

$$u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) = u(c_0) + \beta u(c_1(s)) + A(\triangle) \quad \forall s, \triangle \tag{6}$$

for some $A(\triangle)$, and such that it is resource neutral:

$$-\triangle + q \sum_m \delta(\triangle, s) p(s) = 0 \quad \forall \triangle \tag{7}$$

- With the "free saving" variation, we had $\delta(\triangle, s) = -R^* \triangle$. What is key difference?

VERIFY INCENTIVE COMPATIBILITY OF VARIATION

- Verify that the variation maintains incentive compatibility:

$$u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) - h(y(s)/s) \geq$$

$$u(c_0 - \triangle) + \beta u(c_1(r) + \delta(\triangle, r)) - h(y(r)/s)$$

if and only if (6)

$$u(c_0) + \beta u(c_1(s)) + A(\triangle) - h(y(s)/s) \geq$$

$$u(c_0) + \beta u(c_1(r)) + A(\triangle) - h(y(r)/s)$$

if and only if

$$u(c_0) + \beta u(c_1(s)) - h(y(s)/s) \geq u(c_0) + \beta u(c_1(r)) - h(y(r)/s)$$

- Is this true?
- Key: Given separability, all that matters for incentive compatibility is the total utility from consumption.

- Suppose the original allocation $(c_0, c_1(s), y(s))$ solves the second best problem. Then, since the variation $\delta(\triangle, s)$ is feasible as just shown, it cannot improve the objective.
- Formally,

$$0 = argmax_{\triangle} \sum_s p(s)[u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) - h(y(s)/s)]$$

$$= argmax_{\triangle} \sum_s p(s)[u(c_0) + \beta u(c_1(s) + A(\triangle)) - h(y(s)/s)]$$

$$= argmax_{\triangle} A(\triangle),$$

where we used

$$u(c_0 - \triangle) + \beta u(c_1(s) + \delta(\triangle, s)) = u(c_0) + \beta u(c_1(s)) + A(\triangle) \quad \forall s, \triangle \tag{8}$$

- FOC $A'(0) = 0$

INVERSE EULER EQUATION

- $\delta(\triangle, s)$ satisfies (IC); differentiate w.r.t $\triangle$:

$$-u'(c_0) + \beta u'(c_1(s)) \frac{\partial \delta(\triangle, s)}{\partial \triangle}\Big|_{\triangle=0} = A'(0)$$

rearrange (at optimum):

$$\frac{\partial \delta(\triangle, s)}{\partial \triangle}\Big|_{\triangle=0} = \frac{u'(c_0) + A'(0)}{\beta u'(c_1(s))} = \frac{u'(c_0)}{\beta u'(c_1(s))} \quad \forall s \tag{9}$$

- Condition for resource neutrality of the variation implies:

$$-1 + q \sum_s p(s) \frac{\partial \delta(\triangle, s)}{\partial \triangle}\Big|_{\triangle=0} = 0$$

INVERSE EULER EQUATION (II)

- Using expression for $\left.\frac{\partial\delta(\triangle,s)}{\partial\triangle}\right|_{\triangle=0}$:

$$\frac{1}{u'(c_0)} = \frac{1}{\beta R^*}\sum_s \frac{p(s)}{u'(c_1(s))} \tag{10}$$

- With seperable preferences, optimal allocation has to satisfy this inverse Euler equatoin (Diamond/Mirrlees 1978, Rogerson 1985, Golosov et al. 2003)
- Is this necessary and sufficient? (Think of optimality of $y(s)$).
- Implies that the Euler equation is violated.

$$u'(c_0) = \beta R^* \sum_s p(s) u'(c_1(s)) \tag{11}$$

Is it always violated?

POSITIVE SAVINGS WEDGE

- Inverse Euler implies that, at the optimum,

$$u'(c_0) = \Big[\frac{1}{\beta R^*}\sum_s \frac{p(s)}{u'(c_1(s))}\Big]^{-1} = \beta R^*\Big(\mathbb{E}\Big[\frac{1}{u'(c_1(s))}\Big]\Big)^{-1}$$

  By Jensen's inequality and convexity of the function $f(x) = 1/x$,

$$u'(c_0) < \beta R^* \mathbb{E}[u'(c_1(s))]$$

- The optimal allocation is incompatible with free saving. Is saving is discouraged or encouraged?
- Intuition: saving in period 0 increases income in period 1 across all shocks s $\rightarrow$ negative income effect on y $\rightarrow$ is this good or bad for Planner?
- Implications for capital taxation, but study distinction between the wedge derived here and actual implementations later

TECHNICAL POINT: DUAL APPROACH

- Consider allocation in terms of utils

$$u_0 \equiv u(c_0), \quad u_1(s) \equiv u(c_1(s))$$

- Move from original allocation $(u_0, u_1(s))$ to variation $(\tilde{u}_0, \tilde{u}_1(s))$ such that

$$u_0 + \beta u_1(s) = \tilde{u}_0 + \beta \tilde{u}_1(s) \quad \forall s$$

- In particular, set

$$\tilde{u}_0 = u_0 - \beta \triangle$$

and

$$\tilde{u}_1(s) = u_1(s) + \triangle \quad \forall s$$

TECHNICAL POINT: DUAL APPROACH (II)

- Are incentive constraints affected?

$$\tilde{u_0} + \beta \tilde{u_1}(s) - h(y(s)/s) \geq \tilde{u_1}(r) + \beta \tilde{u_1}(r) - h(y(r)/s) \quad \forall r, s$$

  if and only if

$$u_0 - \beta \triangle + \beta(u_1(s) + \triangle) - h(y(s)/s) \geq$$

$$u_1(r) - \beta \triangle + \beta(u_1(r) + \triangle) - h(y(r)/s) \quad \forall r, s$$

  if and only if

$$u_0 + \beta u_1(s) - h(y(s)/s) \geq u_1(r) + \beta u_1(r) - h(y(r)/s) \quad \forall r, s$$

  is this true?
- Variation by construction keeps total expected utility unchanged
- Dual problem: minimize total resource cost of allocation

$$min_{\triangle} \left\{ C(u_0 - \beta \triangle) + q \sum_{s} p(s) C(u_1(s)) + \triangle) \right\}$$

  where $C(u)$ is the inverse function of $u(c)$

INVERSE EULER AGAIN

- If the original allocation $(u_0, u_1(s))$ is optimal, $\triangle = 0$ must solve this problem
- The FOC evaluated at $\triangle = 0$ is

$$-C'(u_0)\beta + q\sum_s p(s)C'(u_1(s)) = 0$$

- Use $C'(u) = 1/u'(c)$

$$\frac{1}{u'(c_0)} = \frac{q}{\beta}\sum_s \frac{p(s)}{u'(c_1(s))}$$

  which is the inverse Euler equation again (recall $q = 1/R^*$)
- Alternative interpretation: $1/u'(c)$ is resource cost of providing some given incentives
- IEE requires the equalization of the expected resource cost of providing incentives across both periods

INFINITE HORIZON

- General model with separable preferences

$$\sum_{t,s^t} \beta^t [u(c(s^t)) - h(y(s^t)/s_t)] Pr(s^t)$$

  and $s^t = (s_0, s_1, ..., s_t)$
- Agents have reporting strategies such that (why does it depend on s^t not s_t?)

$$r_t = \sigma_t(s^t)$$

  where the truth telling strategy is such that

$$\sigma_t^*(s^t) = s_t \quad \forall s^t, t$$

- $\sigma^t(s^t)$ denotes the history of reports induced by the strategy $\sigma_t(s^t)$, i.e.

$$\sigma^t(s^t) = (r_0, r_1, ..., r_t) = (\sigma_0(s_0), \sigma_1(s^1), ..., \sigma_t(s^t))$$

DYNAMIC INCENTIVE CONSTRAINTS

- Dynamic incentive constraints

$$\sum_{t,s^t} \beta^t [u(c(s^t)) - h(y(s^t)/s_t)/Pr(s^t)$$

$$\geq \sum_{t,s^t} \beta^t [u(c(\sigma^t(s^t))) - h(y(\sigma^t(s^t))/s_t] Pr(s^t) \quad \forall \sigma$$

- Pick some node s^t. Then set

$$\tilde{u}(s^\tau) = u(s^\tau)$$

for any $s^\tau \neq s^t$ and $s^\tau \neq (s^t, s_{t+1})$. i.e. leave consumption utilities unchanged at any node that is not s^t or any of its direct successors

- At s^t, set

$$\tilde{u}(s^t) = u(s^t) - \beta \triangle$$

and

$$\tilde{u}(s^t, s_{t+1}) = u(s^t, s_{t+1}) + \triangle \quad \forall s_{t+1}$$

DYNAMIC INCENTIVE CONSTRAINTS

- Key: if initial allocation was incentive compatible, perturbed one is as well.
- Moreover, perturbed allocation does not change total expected utility (from any reporting strategy $\sigma_t(s^t)$, thus also from truth-telling)
- Minimize expected resource cost of the perturbed allocation by choosing $\triangle$

$$min_{\triangle}\Big\{C(u(s^t)-\beta\triangle)+q\sum_{s^{t+1}|s^t}Pr(s^{t+1}|s^t)C\big(u_1(s^{t+1})+\triangle\big)\Big\}$$

- If the initial allocation is optimal, this program must be solved at $\triangle = 0$ with FOC

$$\frac{1}{u'(c(s^t))}=\frac{1}{\beta R^*}\mathbb{E}\Big[\frac{1}{u'(c(s^{t+1}))}\Big|s^t\Big]$$

General inverse Euler equation has to hold for all nodes s^t

GENERAL INVERSE EULER EQUATION

- Implies

$$u'(c(s^t)) < \beta R^* \mathbb{E}\left[u'(c(s^{t+1}))\middle| s^t\right] \quad \forall s^t$$

i.e. savings need to be distorted downwards compared to the Euler equation from free saving

- May require individualized capital taxes that keep track of the entire history of skill shocks s^t such that

$$u'(c(s^t)) = \beta \mathbb{E}\left[(1 + r^*(1 - \tau^k(s^{t+1})))u'(c(s^{t+1}))\middle| s^t\right] \quad \forall s^t$$

where $r^* \equiv R^* - 1$

- However, simple linear capital tax may not work
- Farhi and Werning (2011) show how to use this framework to evaluate the welfare gains from optimal saving distortions starting from some baseline allocation, e.g. the free saving allocation (Aiyagari 1994)

IMPLEMENTATION

- STEP BACK: What is implementation? Why was it not discussed before?!
- Back to 2 period model, 2 shocks $s \in \{H, L\}$
- Suppose have found the optimal allocation with consumption $\{c_0^*, c_1^*(L), c_1^*(H)\}$
- It satisfies the inverse Euler equation

$$\frac{1}{u'(c_0^*)} = \frac{1}{\beta R^*}\left[\frac{p_L}{u'(c_1^*(L))} + \frac{p_H}{u'(c_1^*(H))}\right]$$

- Suppose we introduce a linear capital tax τ^k such that the Euler equation is satisfied

$$u'(c_0^*) = \beta R^*(1 - \tau^k)[p_H u'(c_1^*(L)) + p_H u'(c_1^*(H))] \tag{12}$$

IMPLEMENTATION: LINEAR CAPITAL TAX, NONLINEAR INCOME TAX

- Introduce a non-linear income tax system $T_0, T_1(y)$ so that the individuals' budget constraints become

$$c_0 + k_1 \leq e_0 - T_0$$

  in period 0 and

$$c_1(s) \leq y(s) - T_1(y(s)) + (1 - \tau^k)R^* k_1$$

  in period 1
- Note: Very restrictive tax system where the capital tax is linear and separable from the labor income tax
- Can we find a tax system T_0, T_1, τ^k such that $\{c_0^*, c_1^*(L), c_1^*(H)\}$ is incentive compatible? [What do you think?]
- If we could force the agent to choose c_0^* and thus k_1^*? We'd be back to a standard Mirlees problem in period 1, so we can always find $T_1(y)$ that implements $c_1^*(s), y^*(s)$

PROBLEM WITH LINEAR CAPITAL TAX

- Suppose H's incentive constraint is the binding one at the optimum (which means?)

$$u(c_1^*(H)) - h(y^*(H)/H) = u(c_1^*(L)) - h(y^*(L)/H) \quad (13)$$

  i.e. *if* the agent saves optimally k_1^*, truth-telling is optimum
- Moreover, *given* truth-telling in period 1, Euler equation holds, so the agent finds it optimal to choose optimal savings k_1^*
- But: double-deviation $\sigma_1(s) = L$ for all $s \in \{H, L\}$ *and* $\tilde{k}_1 = k_1^* + \epsilon$
- If $\sigma_1(s)$ for all s and $k_1 = k_1^*$, then

$$u'(c_0^*) < \beta R^*(1 - \tau^k) u'(c_1^*(L)) \quad (14)$$

  Why?
- It is optimal to deviate to $\tilde{k}_1 = k_1^* + \epsilon$ with $\epsilon > 0$
- What is agent tempted to do here? Explain in words.

DOUBLE DEVIATION

- Profitable deviation: save more in period 0 and always claim to be low type in period 1 period

$$\tilde{U} = u(c_0^* - \epsilon) + \beta\Big[u(c_1^*(L) + R^*(1-\tau^k)\epsilon) - p_L h(y^*(L)/L) - p_H h(y^*(L)/H)\Big]$$

$$\approx \epsilon\Big[-u'(c_0^*) + \beta R^*(1-\tau^k)u'(c_1^*(L))\Big]$$

$$+u(c_0^*) + \beta\Big[u(c_1^*(L)) - p_L h(y^*(L)/L) - p_H h(y^*(L)/(H)\Big]$$

$$> u(c_0^*) + \beta\Big[p_H(u(c_1^*(H)) - h(y^*(H)/H)) + p_L(u(c_1^*(L)) - h(y^*(L)/L))\Big]$$

- Where is first approximation coming from? Where did the second equality come from?
- Hence, the double deviation makes the agent better off than truth telling under the optimal allocation that we wanted to implement.

LINEAR CAPITAL TAX DOES NOT WORK – SOLUTIONS?

- State-dependent linear capital tax $\tau^k(s)$ so that
$$u'(c_0^*) = \beta R^*(1 - \tau^k(s))u'(c_1^*(s)) \quad \forall s$$
state by state (Kocherlakota 2005). Prevents profitable double-deviations.
$$\tau^k(s) = 1 - \frac{u'(c_0^*)}{\beta R^* u'(c_1^*(s))}$$
is high whenever $c_1^*(s)$ is low.
- What does this mean? Returns to saving are made risky so as to make savings unattractive.
- However: $\tau^k(s)$ is zero in expectation so that the government does not raise revenue with the capital tax.
- In general, capital taxes must be contingent on the entire history of shocks.

LINEAR CAPITAL TAX DOES NOT WORK – SOLUTIONS? (II)

- Albanesi and Sleet (2006): joint tax function on wealth and income $T(y, k)$ with iid shocks (wealth is a sufficient statistic for history of shocks with iid shocks)
- Werning (2010): non-linear capital tax works more generally , i.e. $T^k(R^* k_t, s^t)$ rather than $(1 - \tau^k(s^t))R^* k_t$. Moreover, with such a non-linear capital tax, one can make it history-independent, i.e. $T^k(R^* k_t)$, (in contrast to Kocherlakota's implementation).

Adding Human Capital Investments to the Model

- Interplay between HC policies and taxes.
- HC policies affect the income distribution - a key input for taxes.
- Taxes affect return and risk from HC investments.
- Calls for joint analysis of optimal taxation and HC policies.
- Optimal Taxation (Mirrlees) literature typically assumes exogenous ability
  - Mirlees 1971, Saez 2001...

Model: Risky investments in Human Capital

- **Wage**: $w_t = w_t(\theta_t, s_t, z_t)$
- **Ability** θ:
  stochastic, Markov $f^t(\theta_t|\theta_{t-1})$, private info, privately uninsurable.
- Two ways of acquiring HC:
  1. **Expenses** e_t at cost $M_t(e_t)$. Stock of HC expenses s_t :
  $$s_t = s_{t-1} + e_t$$
  2. **Training time** i_t at disutility cost $\phi_t(l_t, i_t)$. Accomplished training z_t :
  $$z_t = z_{t-1} + i_t$$
- Cost composition of College versus OJT?
- Income: $y_t = w_t l_t$

Hicksian complementarity

- **Hicksian coefficients of complementarity**:

$$\rho_{\theta s} = \frac{w_{\theta s} w}{w_s w_\theta} \qquad \rho_{\theta z} = \frac{w_{\theta z} w}{w_z w_\theta}$$

- $\rho_{\theta s} \geq 0$: Marginal wage gain from HC ↑ in ability.

- $\rho_{\theta s} \geq 1$: Elasticity of wage to HC ↑ in ability.

- If separable $w = \theta + h(s, z) \Rightarrow \rho_{\theta s} = \rho_{\theta z} = 0$
- If multiplicative $w = \theta h(s, z) \Rightarrow \rho_{\theta s} = \rho_{\theta z} = 1$
- If CES $w = \left[\alpha_1 \theta^{1-\rho_t} + \alpha_2 s^{1-\rho_t} + \alpha_3 z^{1-\rho_t}\right]^{\frac{1}{1-\rho_t}} \Rightarrow \rho_{\theta s} = \rho_{\theta z} = \rho_t$

▸ Net Subsidy

Model: Preferences over Lifetime Allocations

- T periods of work, T_r periods of retirement.
- Per period utility: $u_t(c_t) - \phi_t(l_t, i_t)$.
- History $\theta^t = \{\theta_1, ..., \theta_t\} \in \Theta^t$, probability $P(\theta^t) = f^t(\theta_{t+1}|\theta_t) ... f(\theta_1)$.
- Allocation: $\{c(\theta^t), y(\theta^t), s(\theta^t), z(\theta^t)\}_{\theta^t}$.
- Expected lifetime utility from allocation:

$$U\left(\left\{c(\theta^t), y(\theta^t), s(\theta^t), z(\theta^t)\right\}\right) = \sum_{t=1}^{T+T_r} \int \beta^{t-1} \left[u_t(c(\theta^t)) - \phi_t\left(\frac{y(\theta^t)}{w_t(\theta^t)}, z(\theta^t) - z\left(\theta^{t-1}\right)\right)\right] P(\theta^t)\, d\theta^t$$

$$w_t(\theta^t) \equiv w_t(\theta_t, s_t(\theta^t), z_t(\theta^t))$$

Government's/Planner's Goals: Insurance and Redistribution

- Govt's/Planner's goal: max expected social welfare given Pareto weights.
  - Insurance against earnings risk.
  - Redistribution across intrinsic ability heterogeneity (persistent).
  - Incentives for efficient work and HC investment.

- Asymmetric information about:

ability and its evolution   labor supply

$$w_t(\theta_t, s_t, z_t) \times l_t = y_t$$

2 cases: observable and unobservable HC.

$\rightarrow$ **"direct revelation mechanism"** with incentive compatibility.

Government's/Planner's Program: Dual Formulation

- Min expected resource cost s.t. utility targets and incentive compatibility $\rightarrow$ **constrained efficiency**.

$$\min_{\{c,y,s,z\}} \sum_{t=1}^{T} \frac{1}{R^{t-1}} \int \left(c\left(\theta^t\right) - y\left(\theta^t\right) + M_t\left(s\left(\theta^t\right) - s\left(\theta^{t-1}\right)\right)\right) P\left(\theta^t\right) d\theta^t$$

$$\text{s.t.: } U\left(\{c, y, s, z\}\right) \geq \underline{U}$$

$$\{c, y, s, z\} \text{ is incentive compatible.}$$

- If initial heterogeneity and non-utilitarian welfare function set any Pareto weights through $\underline{U} = \left(\underline{U}\left(\theta_1\right)\right)_{\Theta}$.

Incentive Compatibility Defined

- Reporting strategy: $r = \{r_t(\theta^t)\}_{t=1}^{T}$, with history $r^t \equiv \{r_1, .., r_t\}$.

- **Continuation utility** under reporting strategy r:

$$\omega^r(\theta^t) = u_t\left(c\left(r^t\left(\theta^t\right)\right)\right) - \phi_t\left(\frac{y\left(r^t\left(\theta^t\right)\right)}{w_t\left(\theta_t, s\left(r^t\left(\theta^t\right)\right), z\left(r^t\left(\theta^t\right)\right)\right)}, i\left(r^t\left(\theta^t\right)\right)\right) + \beta\int \omega^r\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

- **Under truth-telling**: $\omega(\theta^t)$ with $r_t(\theta^t) = \theta_t$ for all θ^t.

- **Incentive Compatibility**

$$\omega(\theta^t) \geq \omega^r(\theta^t) \qquad \forall r, \forall \theta^t$$

Solving the Government's Program: Method

1. Solving the direct revelation mechanism:
   - Step 1: Relax program using first order approach (FOA).
   - Step 2: Formulate relaxed program recursively.
2. Characterize optimal allocations using "wedges" or implicit taxes.
3. Decentralize or "implement" optimum using policy instruments.

Step 1. Relaxing the Program: First-Order Approach

- Consider deviating strategy σ^r with report r:

$$\omega\left(\theta^t\right) = \max_r \Big(u_t\left(c\left(\theta^{t-1}, r\right)\right) - \phi_t\left(\frac{y\left(\theta^{t-1}, r\right)}{w_t\left(\theta_t, s\left(\theta^{t-1}, r\right), z\left(\theta^{t-1}, r\right)\right)}\right) + \beta \int \omega^{\sigma^r}\left(\theta^{t-1}, r, \theta_{t+1}\right) f^{t+1}\left(\theta_{t+1} | \theta_t\right) d\theta_{t+1} \Big)$$

- Replace by necessary **Envelope Condition**:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t} \frac{\partial w_t}{\partial \theta_t} \frac{l_t}{w_t} + \beta \int \omega\left(\theta^{t+1}\right) \frac{\partial f^{t+1}\left(\theta_{t+1} | \theta_t\right)}{\partial \theta_t} d\theta_{t+1}$$

- **Sufficiency**?
  a) Conditions on allocations (Pavan *et al.* 2013).
  b) Ex-post verification (Werning, 2007, Farhi and Werning, 2013).

▸ Verification

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

- Envelope Condition:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t} \frac{\partial w_t}{\partial \theta_t} \frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta \int \omega\left(\theta^{t+1}\right) \frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial \theta_t} d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^{t}\right)=u_{t}\left(c\left(\theta^{t}\right)\right)-\phi_{t}\left(\frac{y\left(\theta^{t}\right)}{w_{t}\left(\theta^{t}\right)}, i\left(\theta^{t}\right)\right)+\beta\int\omega\left(\theta^{t+1}\right)f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)d\theta_{t+1}$$

- Envelope Condition:

$$\frac{\partial\omega\left(\theta^{t}\right)}{\partial\theta_{t}}=\frac{\partial\phi_{t}}{\partial l_{t}}\frac{\partial w_{t}}{\partial\theta_{t}}\frac{l\left(\theta^{t}\right)}{w\left(\theta^{t}\right)}+\beta\int\omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)}{\partial\theta_{t}}d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta v_t\left(\theta^t\right)$$

- Envelope Condition:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t}\frac{\partial w_t}{\partial \theta_t}\frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta \int \omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial \theta_t} d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^{t}\right)=u_{t}\left(c\left(\theta^{t}\right)\right)-\phi_{t}\left(\frac{y\left(\theta^{t}\right)}{w_{t}\left(\theta^{t}\right)},i\left(\theta^{t}\right)\right)+\beta v_{t}\left(\theta^{t}\right)$$

- Envelope Condition:

$$\frac{\partial\omega\left(\theta^{t}\right)}{\partial\theta_{t}}=\frac{\partial\phi_{t}}{\partial l_{t}}\frac{\partial w_{t}}{\partial\theta_{t}}\frac{l\left(\theta^{t}\right)}{w\left(\theta^{t}\right)}+\beta\int\omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)}{\partial\theta_{t}}d\theta_{t+1}$$

$$v_{t}\left(\theta^{t}\right)=\int\omega\left(\theta^{t+1}\right)f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^{t}\right)=u_{t}\left(c\left(\theta^{t}\right)\right)-\phi_{t}\left(\frac{y\left(\theta^{t}\right)}{w_{t}\left(\theta^{t}\right)},i\left(\theta^{t}\right)\right)+\beta v_{t}\left(\theta^{t}\right)$$

- Envelope Condition:

$$\frac{\partial\omega\left(\theta^{t}\right)}{\partial\theta_{t}}=\frac{\partial\phi_{t}}{\partial l_{t}}\frac{\partial w_{t}}{\partial\theta_{t}}\frac{l\left(\theta^{t}\right)}{w\left(\theta^{t}\right)}+\beta\int\omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)}{\partial\theta_{t}}d\theta_{t+1}$$

$$v_{t}\left(\theta^{t}\right)=\int\omega\left(\theta^{t+1}\right)f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^{t}\right)=u_{t}\left(c\left(\theta^{t}\right)\right)-\phi_{t}\left(\frac{y\left(\theta^{t}\right)}{w_{t}\left(\theta^{t}\right)},i\left(\theta^{t}\right)\right)+\beta v_{t}\left(\theta^{t}\right)$$

- Envelope Condition:

$$\frac{\partial\omega\left(\theta^{t}\right)}{\partial\theta_{t}}=\frac{\partial\phi_{t}}{\partial l_{t}}\frac{\partial w_{t}}{\partial\theta_{t}}\frac{l\left(\theta^{t}\right)}{w\left(\theta^{t}\right)}+\beta\Delta_{t}\left(\theta^{t}\right)$$

$$v_{t}\left(\theta^{t}\right)=\int\omega\left(\theta^{t+1}\right)f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^{t}\right)=u_{t}\left(c\left(\theta^{t}\right)\right)-\phi_{t}\left(\frac{y\left(\theta^{t}\right)}{w_{t}\left(\theta^{t}\right)},i\left(\theta^{t}\right)\right)+\beta v_{t}\left(\theta^{t}\right)$$

- Envelope Condition:

$$\frac{\partial\omega\left(\theta^{t}\right)}{\partial\theta_{t}}=\frac{\partial\phi_{t}}{\partial l_{t}}\frac{\partial w_{t}}{\partial\theta_{t}}\frac{l\left(\theta^{t}\right)}{w\left(\theta^{t}\right)}+\beta\Delta_{t}\left(\theta^{t}\right)$$

$$\begin{aligned} v_{t}\left(\theta^{t}\right) &= \int\omega\left(\theta^{t+1}\right)f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)d\theta_{t+1} \\ \Delta_{t}\left(\theta^{t}\right) &= \int\omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_{t}\right)}{\partial\theta_{t}}d\theta_{t+1} \end{aligned}$$

Step 2. Recursive Formulation: Rewrite Recursively

- Definition of continuation utility, using $\theta_-, \theta, \theta'$.

$$\omega(\theta) = u_t(c(\theta)) - \phi_t\left(\frac{y(\theta)}{w_t(\theta)}, z(\theta) - z_-\right) + \beta v(\theta)$$

- Envelope Condition:

$$\frac{\partial \omega(\theta)}{\partial \theta} = \frac{\partial \phi_t}{\partial l}\frac{\partial w_t(\theta)}{\partial \theta}\frac{l(\theta)}{w_t(\theta)} + \beta \Delta(\theta)$$

$$v(\theta) = \int \omega(\theta') f^{t+1}(\theta'|\theta)\, d\theta'$$

$$\Delta(\theta) = \int \omega(\theta') \frac{\partial f^{t+1}(\theta'|\theta)}{\partial \theta} d\theta'$$

Recursive Formulation of Relaxed Program

$$K\left(v,\Delta,\theta_{-},s_{-},z_{-},t\right) = \min \int \left(c\left(\theta\right) + M_t\left(s\left(\theta\right) - s_{-}\right) - w_t\left(\theta, s\left(\theta\right), z\left(\theta\right)\right) l\left(\theta\right) + \frac{1}{R} K\left(v\left(\theta\right), \Delta\left(\theta\right), \theta, s\left(\theta\right), z\left(\theta\right), t+1\right)\right) f^t\left(\theta|\theta_{-}\right) d\theta$$

$$\begin{aligned}
\omega\left(\theta\right) &= u_t\left(c\left(\theta\right)\right) - \phi_t\left(l\left(\theta\right), z\left(\theta\right) - z_{-}\right) + \beta v\left(\theta\right) \\
\dot{\omega}\left(\theta\right) &= \frac{w_{\theta,t}}{w_t} l\left(\theta\right) \phi_{l,t}\left(\left(\theta\right), z\left(\theta\right) - z_{-}\right) + \beta \Delta\left(\theta\right) \\
v &= \int \omega\left(\theta\right) f^t\left(\theta|\theta_{-}\right) d\theta \\
\Delta &= \int \omega\left(\theta\right) \frac{\partial f^t\left(\theta|\theta_{-}\right)}{\partial \theta_{-}} d\theta
\end{aligned}$$

over $\left(c\left(\theta\right), l\left(\theta\right), s\left(\theta\right), z\left(\theta\right), \omega\left(\theta\right), v\left(\theta\right), \Delta\left(\theta\right)\right)$

Recursive Formulation of Relaxed Program

$$K(v,\Delta,\theta_-,s_-,z_-,t) = \min \int \Big(c(\theta) + M_t(s(\theta)-s_-) - w_t(\theta,s(\theta),z(\theta))\,l(\theta) + \frac{1}{R}K(v(\theta),\Delta(\theta),\theta,s(\theta),z(\theta),t+1)\Big) f^t(\theta|\theta_-)\,d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta),z(\theta)-z_-) + \beta v(\theta)$$

$$\dot{\omega}(\theta) = \frac{w_{\theta,t}}{w_t} l(\theta)\,\phi_{l,t}((\theta),z(\theta)-z_-) + \beta\Delta(\theta)$$

$$v = \int \omega(\theta)\, f^t(\theta|\theta_-)\,d\theta$$

$$\Delta = \int \omega(\theta)\,\frac{\partial f^t(\theta|\theta_-)}{\partial\theta_-}\,d\theta$$

over $(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta))$

Recursive Formulation of Relaxed Program

$$
\begin{aligned}
K\left(v,\Delta,\theta_-,s_-,z_-,t\right) = \min \int \Big(& c(\theta) + M_t\left(s(\theta)-s_-\right) - w_t\left(\theta, s(\theta), z(\theta)\right) l(\theta) \\
& + \frac{1}{R} K\left(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1\right)\Big) f^t\left(\theta|\theta_-\right) d\theta
\end{aligned}
$$

$$
\begin{aligned}
\omega(\theta) &= u_t\left(c(\theta)\right) - \phi_t\left(l(\theta), z(\theta)-z_-\right) + \beta v(\theta) \\
\dot{\omega}(\theta) &= \frac{w_{\theta,t}}{w_t} l(\theta)\, \phi_{l,t}\left((\theta), z(\theta)-z_-\right) + \beta \Delta(\theta) \\
v &= \int \omega(\theta) f^t\left(\theta|\theta_-\right) d\theta \\
\Delta &= \int \omega(\theta) \frac{\partial f^t\left(\theta|\theta_-\right)}{\partial \theta_-} d\theta
\end{aligned}
$$

over $\left(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta)\right)$

Recursive Formulation of Relaxed Program

$$K(v,\Delta,\theta_-,s_-,z_-,t) = \min \int \Big(c(\theta) + M_t(s(\theta)-s_-) - w_t(\theta, s(\theta), z(\theta))\, l(\theta) + \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1)\Big) f^t(\theta|\theta_-)\, d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta), z(\theta)-z_-) + \beta v(\theta)$$

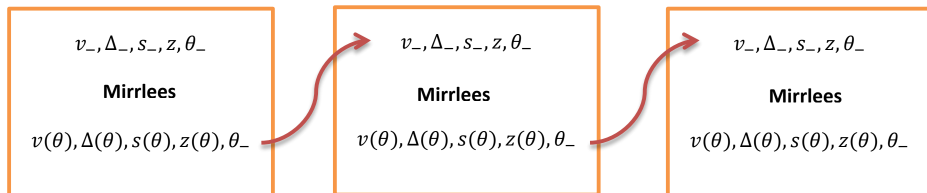
$$\dot{\omega}(\theta) = \frac{w_{\theta,t}}{w_t} l(\theta)\, \phi_{l,t}((\theta), z(\theta)-z_-) + \beta\Delta(\theta)$$

$$v = \int \omega(\theta)\, f^t(\theta|\theta_-)\, d\theta$$

$$\Delta = \int \omega(\theta) \frac{\partial f^t(\theta|\theta_-)}{\partial\theta_-} d\theta$$

over $(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta))$

Method summary:
Repeated Mirrlees Nested in Dynamic Programming
+ Endogenous Human Capital Formation



Outline

1. Model and Solution Approach
2. Human Capital Expenses
3. Training Time
4. Unobservable Human Capital
5. Implementation

Implicit taxes and subsidies: Definition

Implicit marginal labor income tax:

$$\tau_{Lt} \equiv 1 - \underbrace{\frac{\phi_{l,t}\left(l_t, i_t\right)}{w_t u_t'\left(c_t\right)}}_{\text{MRS/MRT } l_t \text{ and } c_t}$$

Implicit marginal savings tax:

$$\tau_{kt} \equiv 1 - \underbrace{\frac{1}{\beta R} \frac{u_t'\left(c_t\right)}{E_t\left(u_{t+1}'\left(c_{t+1}\right)\right)}}_{\text{(MRS } c_t \text{ and } c_{t+1}\text{)/Return on savings}}$$

Implicit marginal HC subsidy:

$$\tau_{St} \equiv \underbrace{M_t'\left(e_t\right) - \beta E_t\left(\frac{u_{t+1}'\left(c_{t+1}\right)}{u_t'\left(c_t\right)} M_{t+1}'\left(e_{t+1}\right)\right)}_{\text{Dynamic (risk-adjusted) Cost}} - \underbrace{\left(1 - \tau_{Lt}\right) w_{s,t} l_t}_{\text{Marginal Benefit}}$$

Implicit marginal bonus for training time:

$$\tau_{Zt} \equiv \underbrace{\frac{\phi_{i,t}\left(l_t, i_t\right)}{u_t'\left(c_t\right)} - \beta E_t\left(\frac{u_{t+1}'}{u_t'} \frac{\phi_{i,t+1}\left(l_{t+1}, i_{t+1}\right)}{u_{t+1}'\left(c_{t+1}\right)}\right)}_{\text{Dynamic (risk-adjusted, monetary) cost}} - \underbrace{\left(1 - \tau_{Lt}\right) w_{z,t} l_t}_{\text{Marginal Benefit}}$$

A Net Human Capital Subsidy to Capture Real Incentives

Definition (Net Wedge)

$$t_{st} \equiv \frac{\tau_{St} - \tau_{Lt} M_t'^d + P_t}{\left(M_t'^d - \tau_{St}\right)\left(1 - \tau_{Lt}\right)}$$

$M_t'^d \equiv M_t' - \beta E_t \left(\frac{u_{t+1}'}{u_t'} M_{t+1}' \right)$: risk adjusted dynamic cost

$P_t \equiv \frac{1}{R} \frac{\tau_k}{1-\tau_k} \left(1 - \tau_{Lt}\right) E \left(\beta \frac{u_{t+1}'}{u_t'} M_{t+1}' \right)$: risk adjusted savings distortion

$t_{st} = 0 \rightarrow$ **Full dynamic risk-adjusted deductibility of expenses**
Neutrality of tax system wrt HC.

Static model: $\tau_{St}^* = M_t' \tau_{Lt}^*$, standard deductibility (BJ, 2005).

Dynamic model + uncertainty: $\tau_{St}^* = M_t'^d \tau_{Lt}^* - P_t^*$
i) dynamic cost, ii) risk adjustment, iii) savings wedge.

$t_{st} > 0 \rightarrow$ positive net subsidy beyond deductibility.

Optimal Net Subsidy: the Formula

$$t_{st}^{*}\left(\theta^{t}\right)=\frac{\left(\kappa\left(\theta^{t}\right)+\eta\left(\theta^{t}\right)\right)u_{t}'\left(c\left(\theta^{t}\right)\right)}{f^{t}\left(\theta_{t}|\theta_{t-1}\right)}\frac{\varepsilon_{w\theta,t}}{\theta_{t}}\left(1-\rho_{\theta s,t}\right)$$

$\epsilon_{w\theta,t}$: wage elasticity wrt ability.

1. Insurance Motive

$$t_{st}^{*}\left(\theta^{t}\right)=\frac{\left(\kappa\left(\theta^{t}\right)+\eta\left(\theta^{t}\right)\right)u_{t}'\left(c\left(\theta^{t}\right)\right)}{f^{t}\left(\theta_{t}|\theta_{t-1}\right)}\frac{\varepsilon_{w\theta,t}}{\theta_{t}}\left(1-\rho_{\theta s,t}\right)$$

$$\kappa\left(\theta^{t}\right)\equiv\int_{\theta_{t}}^{\bar{\theta}}\left(\frac{1}{u_{t}'\left(c\left(\theta^{t-1},\theta_{s}\right)\right)}-E_{t-1}\left(\frac{1}{u_{t}'\left(c\left(\theta^{t}\right)\right)}\right)\right)f\left(\theta_{s}|\theta_{t-1}\right)d\theta_{s}$$

Insurance Motive

$\kappa\left(\theta^{t}\right)$ captures dispersion in marginal utilities

$\kappa\left(\theta^{t}\right)=0$ if quasilinear utility or no uncertainty (fully persistent types).

2. Persistence and the Redistributive Motive

$$t^*_{st}\left(\theta^t\right) = \frac{\left(\kappa\left(\theta^t\right) + \eta\left(\theta^t\right)\right) u'_t\left(c\left(\theta^t\right)\right)}{f^t\left(\theta_t|\theta_{t-1}\right)} \frac{\varepsilon_{w\theta,t}}{\theta_t}\left(1-\rho_{\theta s,t}\right)$$

$$\eta\left(\theta^t\right) \equiv t^*_{st-1}\left(\theta^{t-1}\right)\left[\frac{R\beta}{u'_{t-1}} \frac{1}{\left(1-\rho_{\theta s,t-1}\right)} \frac{\theta_{t-1}}{\varepsilon_{w\theta,t-1}} \int_{\theta_t}^{\bar{\theta}} \frac{\partial f\left(\theta_s|\theta_{t-1}\right)}{\partial \theta_{t-1}} d\theta_s\right]$$

Persistence and the Redistributive Motive

Persistence of ability $\rightarrow$ persistence of policy.

$\eta\left(\theta^t\right) = 0$ with iid shocks.

Redistributive motive against initial heterogeneity remains active if persistence.

▸ Initial heterogeneity

3. Complementarity Between HC and Ability

$$t_{st}^{*}\left(\theta^{t}\right)=\frac{\left(\kappa\left(\theta^{t}\right)+\eta\left(\theta^{t}\right)\right)u_{t}'\left(c\left(\theta^{t}\right)\right)}{f^{t}\left(\theta_{t}|\theta_{t-1}\right)}\frac{\varepsilon_{w\theta,t}}{\theta_{t}}\left(1-\rho_{\theta s,t}\right)$$

$$t_{st}^{*}\left(\theta^{t}\right)\geq 0\Leftrightarrow\rho_{\theta s,t}\leq 1$$

Labor Supply Effect:
subsidy increases wage
$\rightarrow\uparrow$ labor
$\rightarrow\uparrow$ resources.

$+$

Inequality Effect:
if $\rho_{\theta s}\geq 0$
HC benefits more able agents more
$\rightarrow\uparrow$ *pre-tax* inequality and risk.

$\rho_{\theta s}\leq 1\Rightarrow$ subsidy $\downarrow$ *post-tax* inequality
$\Rightarrow$ has **positive redistributive and insurance effects**.

$\rho_{\theta s}=1\Rightarrow t_{st}^{*}\left(\theta^{t}\right)=0$
Benchmark case in literature $w_t=\theta_t s_t$
Full dynamic risk-adjusted deductibility $\approx$ Atkinson-Stigliz result.

$\rho_{\theta s}$

Evolution of the net subsidy over time

- If $\log(\theta_t) = p \log(\theta_{t-1}) + \psi_t$, with $f^{\psi}\left(\psi|\theta^{t-1}\right)$ and $E\left(\psi|\theta^{t-1}\right) = 0$.

$$E_{t-1}\left(t_{st}\frac{\varepsilon_{w\theta,t-1}}{\varepsilon_{w\theta,t}}\frac{(1-\rho_{\theta s,t-1})}{(1-\rho_{\theta s,t})}\left(\frac{1}{R\beta}\frac{u'_{t-1}}{u'_t}\right)\right)$$
$$= (1-\rho_{\theta s,t-1})\,\varepsilon_{w\theta,t-1}Cov\left(\frac{1}{R\beta}\frac{u'_{t-1}}{u'_t}, \log(\theta_t)\right) + pt_{st-1}$$

- If HC has **positive insurance value** ($\rho_{\theta s} \leq 1$): positive drift.
  - Fading drift term $\rightarrow$ "**Subsidy smoothing.**"

- Persistence of shocks $\rightarrow$ persistence of policy t_{st}.

Empirical Estimates of the Hicksian Complementarity

- **Labor/Human Capital Literature**:
  Heckman, Cunha et al., 2006, Ashenfelter and Rouse, 1998, ...
  - Early Childhood investments level playing field $\Rightarrow \rho_{\theta s} \leq 1$.
  - Evidence suggests $\rho_{\theta s}$ changes over life.
  - College benefits already able people $\Rightarrow \rho_{\theta s} \geq 0$ and $\rho_{\theta s} \geq 1$ possible.
- **Structural Macro Literature**:
  Huggett, Ventura, Yaron, 2010, Heathcote, Perri, Violante, 2010:
  - Ability as the residual, assume log separability $\Leftrightarrow \rho_{\theta s} = 1$.
- **OJT**? Investments later in life? Scarce empirical evidence.

Sufficient Stats

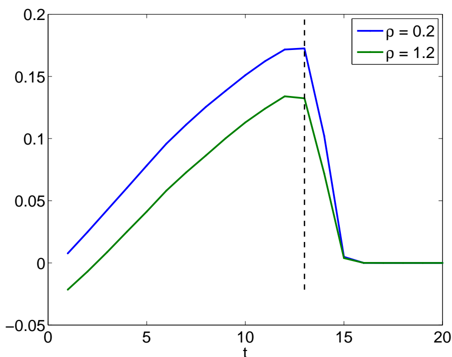
Numerical Analysis: Setup

	Functional Form	Values	Source
Wage	$w_t = \left(\theta_t^{1-\rho} + c_s s_t^{1-\rho}\right)^{\frac{1}{1-\rho}}$	$\rho = \{0.2, 1.2\}$ $c_s = \{0.09, 0.1\}$	CHLM (2006) Match wage premium (AKK, 1998)
Utility	$\log(c_t) - \frac{1}{\gamma}\left(\frac{y_t}{w_t}\right)^{\gamma}$	$\gamma = 3$	Chetty (2012)
Stochastic process	$\log\theta_t = \log\theta_{t-1} + \psi_t$ $\psi_t \sim N\left(-\frac{1}{2}\sigma_\psi^2, \sigma_\psi^2\right)$	$\sigma_\psi^2 = 0.0095$	HSV (2005)
Cost	$M_t(e_t) = c_l e_t + 2\left(\frac{e_t}{s_{t-1}}\right)^2$	$c_l = 0.5$	Match expenses OECD (2013) US DoE (2010)

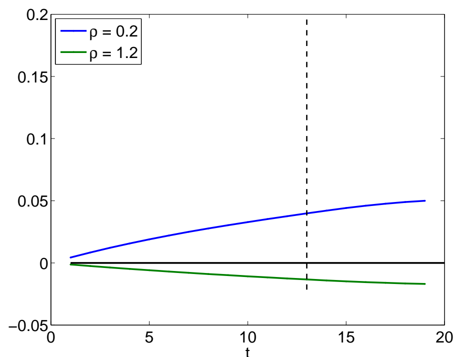
table 1

- $T = 20,\ T_r = 10,\ \beta = 0.95,\ R = 1/\beta$.
- Select zero net cost allocation, utilitarian planner. ▸ Match ▸ Baseline ▸ Vol

Optimal Gross and Net Human Capital Wedges



(a) Gross Wedge τ_{St}



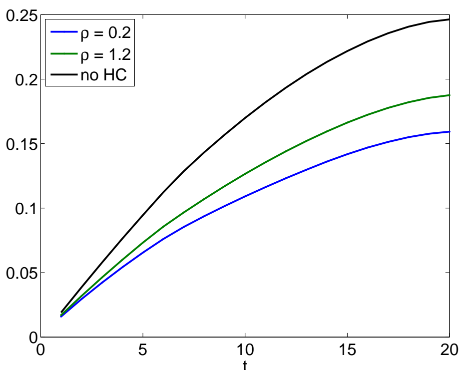
(b) Net Wedge t_{st}

If $\rho_{\theta s} < 1$, τ_S higher and grows faster; $t_s > 0$ and growing.

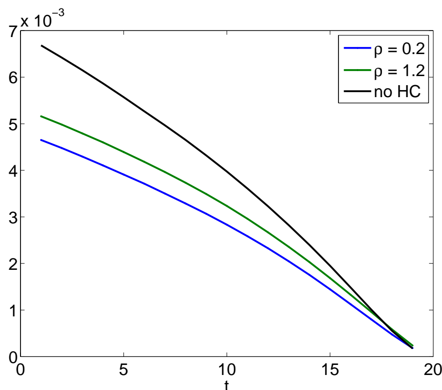
But: Full dynamic risk adjusted deductibility close to optimal.

▸ Effects of $\rho_{\theta s}$ ▸ Effects of volatility

Labor and Capital Wedges with Human Capital



(a) Labor Wedge over time



(b) Capital Wedge over time

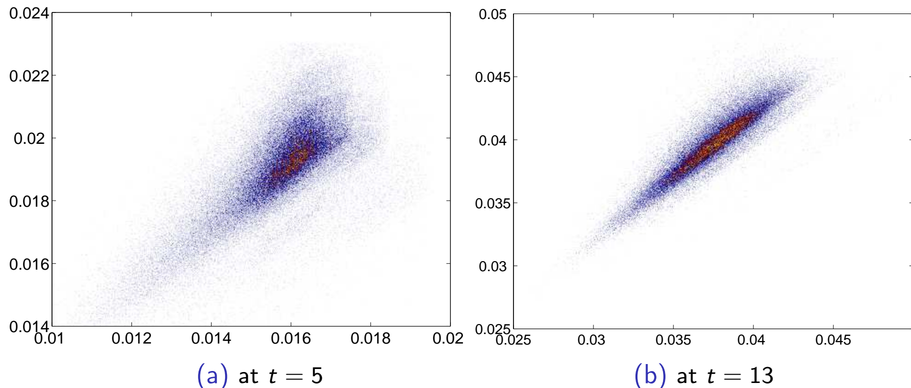
Labor and capital wedges are smaller in the presence of HC.

Standard Inverse Euler Equation holds.

Tax Regressivity
Variance Cons Growth

Subsidy Smoothing over Life

Figure: t_{st} against t_{st-1}



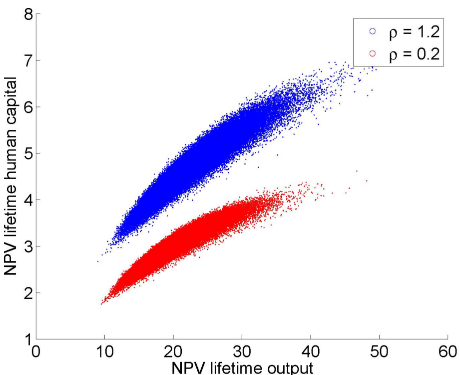
(a) at $t = 5$

(b) at $t = 13$

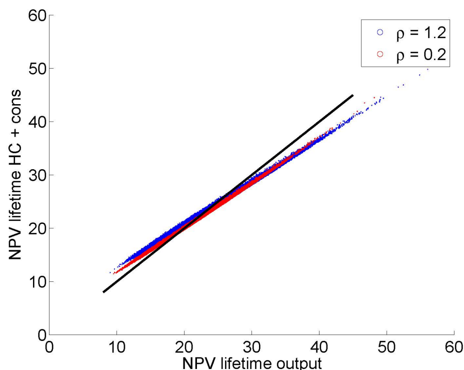
t_s becomes more correlated over time as age increases because the variance of consumption growth vanishes.

▸ Subsidy Regressivity ▸ Tax Smoothing

Insurance and HC Over the Life Cycle



(a) HC against lifetime income



(b) Consumption + HC against lifetime income

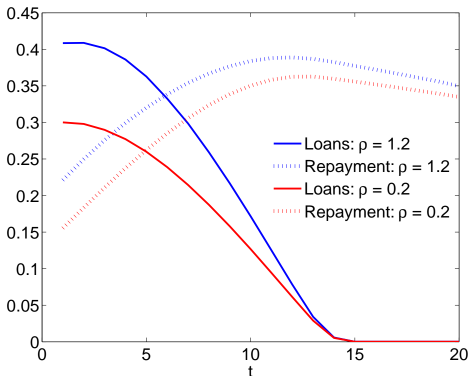
▸ Allocations

Implementation: Idea

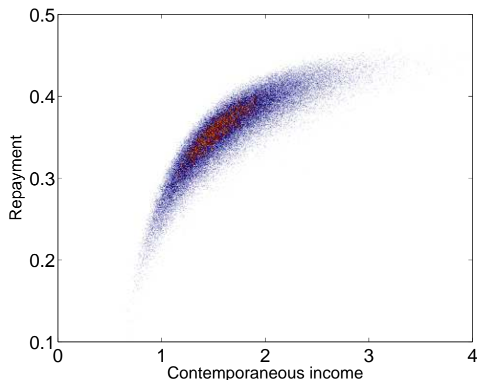
- From direct revelation mechanism to policy instruments.
- "Taxation principle" $\rightarrow$ $T_t\left(y^t, s^t\right)$ implements optimum.
- Indeterminacy of instruments in theory:
  Administrative constraints or political preferences in practice?
- Propose ICLs and Deferred Deductibility Scheme.

Income Contingent Loans (ICLs)

- Loan covers HC cost: $L_t(e_t) = M_t(e_t)$.
- Repayment based on history of loans and earnings: $R_t\left(L^{t-1}, y^{t-1}, e_t, y_t\right)$.



(a) Loans and repayment as % of income



(b) Insurance role and history contingency of repayments

Agent's program

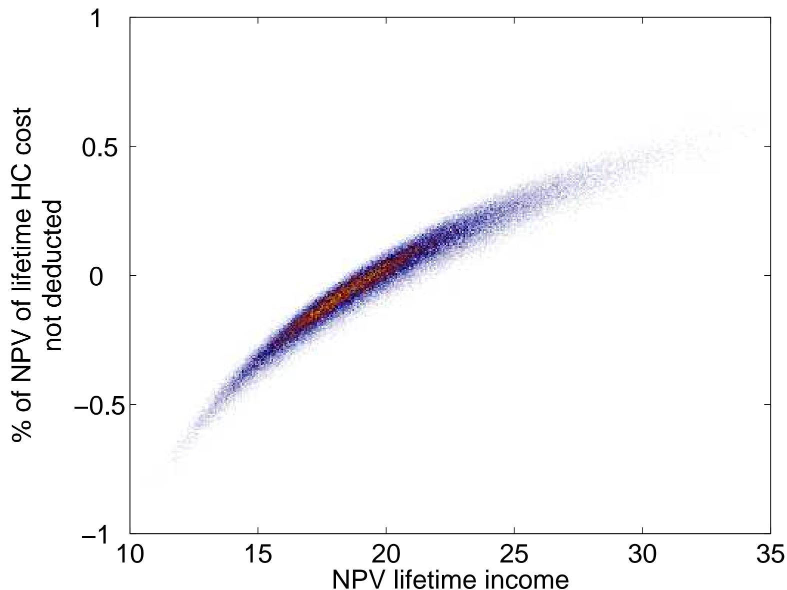
Simplified versions of ICLs exist

- Proposed by J. Tobin and M. Friedman.
- Tried in Sweden, Australia, NZ, UK, US, Chile, SA, Thailand.
  - US: ICR for public sector (1994), IBR since 2007 (CRRAA).
  - Australia: HECS - automatic, collected by tax authority.
- Main differences of scheme proposed here:
  - Not only for College
  - Longer history-dependence
  - Focus on both downside and upside.
- "Yale Plan" debacle (1970s): need tax power of govt (adverse selection).

Deferred Deductibility Scheme

- Part of expense made at t deducted at $t+j$.
  - Nonlinear cost: deduct at MC effective at $t+j$, not historic MC. ▸ Nonlinear
  - Linear cost: $(1-\beta)\%$ of cost. ▸ Linear
  - Plus "no arbitrage" for physical capital taxation.
- Not sufficient to make HC expenses **contemporaneously deductible**:
  - Changing nonlinear tax rates
  - Savings tax
  - Risk adjustment (varying u').

Out-of-pocket HC costs at different income levels



What welfare gain can simpler policies achieve?

Policy studied:
Set linear $\tau_{Lt}, \tau_{St}, \tau_{Kt}$ to cross-sectional average (across all histories θ^t at age t).

Table 2: Welfare Gains

	$\rho_{\theta s} = 0.2$		$\rho_{\theta s} = 1.2$	
Volatility	Medium	High	Medium	High
Welfare gain from second best	0.85%	1.60%	0.98%	1.76%
Welfare gain from linear				
age-dependent policies	0.79%	1.53%	0.94%	1.74%
as % of second best	93%	95.6%	95.5%	98.5%

Conclusion

- Applications: entrepreneurial taxation, bequest taxation, health care?

- Open empirical questions:
  i) Estimate $\rho_{\theta s}$ (also later in life investments).
  ii) How strongly does HC respond to taxes (weaker: to net returns)?

Bottomline

- Crucial consideration: complementarity of HC to ability and risk + direct interaction with labor time.

- Numerically: i) Full dynamic risk-adjusted deductibility close to optimal.
  ii) Simpler age-dependent linear policies perform very well.

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