Economics 2450A: Public Economics
Section 2: Introduction to Optimal Income Taxation

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In today's section we will introduce the problem of optimal income taxation. We will set up the government problem and derive optimal taxes. We will study optimal linear tax rate, optimal top tax rate and the revenue maximizing tax rate.

1 The Income Taxation Problem

Our goal for most of this class is to derive the properties of optimal taxes in different context. We will define the tax in a flexible way using the mathematical object $T(z)$, where z is the income reported by the agent. The tax $T(z)$ generates the *retention function* $R(z) = z - T(z)$. $R(z)$ measures how much the agent can retain out of total income z. We denote transfers to income z with $-T(z)$ so that the transfer $-T(0)$ to non-working individuals is the intercept of the retention function.

If $T(z)$ is differentiable, $T'(z)$ represents the *marginal tax rate*. It measures how much the agent gets taxed out of one additional dollar of income.

In order to study the extensive margin decision between working and remaining unemployed, we need to know the *participation tax rate* $\tau_p = \frac{T(z)-T(0)}{z}$. It is the fraction of income that an agent pays in taxes when she moves from 0 income to z.

2 Taxation in a Model With No Behavioral Responses

We start with a simple version of an optimal income taxation problem that ignores the labor supply response to taxation. Suppose the agent has utility $u(c)$ such that $u'(c) > 0$ and $u''(c) \leq 0$. Labor does not enter the utility function and it is supplied inelastically. The agent consumes everything that is left after taxes so that $c = z - T(z)$. The economy is populated by several agents and their income is distributed according to $h(z)$ with support $[0, \infty]$.

We study the problem of a government, whose goal is to maximize the total utility of the economy. Every agent in the economy is equally weighted such that:

$$\int_0^\infty u(z - T(z))\, h(z)\, dz$$

We call this type of social welfare function *utilitarian*. The government targets a level of revenues E and its budget constraint is:

$$\int_0^\infty T(z)\, h(z)\, dz \geq E$$

The Lagrangian for the problem reads:

$$L = [u(z - T(z)) + \lambda T(z)]\, h(z)$$

Where λ is constant across individuals and measures the value of government revenues in equilibrium. The optimal choice of $T(z)$ delivers the following first order condition:

$$\frac{\partial L}{\partial T(z)} = [-u'(z - T(z)) + \lambda] h(z) = 0$$

Rearranging:

$$u'(z - T(z)) = \lambda$$

Notice that since λ is constant and all agents have the same preferences, the equilibrium condition implies that consumption is equalized across all individuals. This is a direct consequence of the utilitarian social welfare function and the concavity of the utility. Suppose that we taxed a rich individual who would otherwise have a high level of consumption to redistribute to a poor who would otherwise have low consumption. The marginal utility gain of the poor would be higher that the marginal utility loss of the rich if the utility has decreasing marginal returns (implied by the concavity of the utility function). This implies that until all consumption levels are equalized across the economy the government can increase social welfare through "redistribution" from rich to poor individuals. Since every agent has the same weight in the government social welfare function, the optimal policy will treat all individuals equally. There is no gain for the government from guaranteeing a higher level of consumption to a particular group of individuals.

Taxes will serve the purpose of collecting the revenues needed to meet the requirement E. Each individual consumes $c = \bar{z} - E$, where $\bar{z} = \int_0^\infty z h(z)\, dz$ is the average income. Therefore, we have a 100% marginal tax rate above $\tilde{z} = \bar{z} - E$.

3 Towards the Mirrlees Optimal Income Tax Model

The main limitation of the model presented in the previous paragraph is the absence of behavioral responses. Agents were not allowed to respond to fiscal incentives and adjust the labor supply according to the tax schedule. We showed that an extreme case of 100% marginal tax rate can be optimal without causing a loss of revenues due to lower labor supply. We now relax the assumption of inelastic labor supply and study a more flexible model.

Suppose the agent has preferences over consumption and labor represented by the utility function $u(c, l)$. Each agent earns income wl when supplying l hours of labor and consumes $c = wl - T(wl)$ after taxes. Individuals are heterogeneous in the salary w that we will interpret as a measure of ability. Salaries are distributed according to $f(w)$.

Changes in taxes have labor supply effects that depends on the characteristics of the change. A lump-sum change in the level of taxes at a given income changes labor supply through an income effect. On the other hand, a shift in the marginal tax rate causes a distortion in the labor supply through a substitution effect.

Social Welfare Functions: The general problem in Mirrlees (1971) assumes that individual welfare is aggregated through a social welfare function $G(\cdot)$. We typically assume that $G(\cdot)$ is concave in order to represent redistributive preferences of the government. We define the following a social marginal welfare weight:

$$g_i = \frac{G'\left(u^i\right) u_c^i}{\lambda}$$

It measures the government marginal utility from giving a dollar to individual i. The expression is scaled by the marginal value of revenues to the government (λ), that converts the marginal utility in money metric. The concavity of the utility implies that g_i is decreasing in z_i. The social welfare effect of giving \$1 to all the individuals in the economy is therefore $\int_i g_i$.

4 Optimal Linear Tax Rate

In this paragraph we study the optimal income tax when we restrict the instruments that the government can use to tax income. We focus on *linear taxes* τ. The revenues of the tax are rebated through lump-sum transfers. The individual therefore consumes:

$$c_i = (1-\tau)\, w_i l_i + \tau Z$$

where Z represents the total income level in equilibrium and therefore τZ is the total tax revenue from the tax.

The government sets the linear tax to maximize the following:

$$\int_i G\left[u_i\left((1-\tau)\, w_i l_i + \tau Z, l_i\right)\right]$$

Notice that we do not have any government budget constraint since the entire revenue is rebated.

Applying the Envelope theorem we get:

$$\begin{aligned} \int_i G'(u_i)\, u_i' \left[-w_i l_i + Z - \tau \frac{dZ}{d(1-\tau)}\right] &= 0 \\ \int_i G'(u_i)\, u_i' \left[-z_i + Z - \frac{\tau}{(1-\tau)} Z \varepsilon_{z,1-\tau}\right] &= 0 \end{aligned}$$

Where the second line exploits the definition of uncompensated elasticity. Unlike z_i, we implicitely differentiate Z since the individual does not maximize over Z, but takes the transfer as given. In other words the agent does not internalize the effect of her labor supply choice on aggregate revenues and tranfers. This is why the Envelope theorem does not apply to Z.

The two terms in the expression above are central in the optimal taxation literature:

- $Z - z_i$ is the mechanical effect of the tax. Suppose we keep labor supply unchanged, an increase in τ generates a drop in income of z_i and a mechanical increase in transfers of Z due to higher revenues.

- $\frac{\tau}{(1-\tau)} Z \varepsilon_{z,1-\tau}$ is the behavioral effect of the tax. If we allow individuals to adjust their labor supplies we have to take into account the fiscal externality on revenues: when people work less the government collects lower revennues.

We could expect to see in the formula the utility consequence of a change in labor supply. However, any welfare effect related to the behavioral response of the individual is excluded. The reason is that although the agent changes the labor supply, if the tax change is small enough we can neglet the utility effect invoking the envelope theorem. Remember that the logic of the envelope theorem is that after we shift a parameter (the tax in this case) the agent is moving to a new bundle on the same indifference curve.

Rearranging the optimality condition we find:

$$\begin{aligned} Z \int_i g_i - \int_i g_i z_i &= \frac{\tau}{(1-\tau)} Z \varepsilon_{z,1-\tau} \int_i g_i \\ 1 - \frac{\int_i g_i z_i}{Z \int_i g_i} &= \frac{\tau}{(1-\tau)} \varepsilon_{z,1-\tau} \end{aligned}$$

We define $\bar{g} = \frac{\int_i g_i z_i}{Z \int_i g_i}$ and rewrite the condition above to get the optimal tax rate:

$$\tau^* = \frac{1-\bar{g}}{1-\bar{g}+\varepsilon_{z,1-\tau}}$$

The optimal tax is decreasing in $\varepsilon_{z,1-\tau}$ and $\bar{g}$. When income is very elastic to taxes, the government will tax less to avoid negative effects on revenues and transfers coming from distortions to the labor supply. This is the efficiency part of the formula. On the other hand, $\bar{g}$ is a measure of inequality in the economy. It is low when income is extremely polarized. Therefore, the government increases taxes at the optimum when inequality is high. This is the equity part of the formula.

5 Optimal Top Income Taxation

We now derive taxes as in Saez (2001). Instead of specifying a model, we consider the different effects of a tax change and derive the tax by imposing that their sum is zero in equilibirum. Suppose the government wants to optimally set a constant marginal tax rate τ above an income threshold z^*. The average income above z^* is denoted by $z\,(1-\tau)$ and it depends on the tax rate in place. The uncompensated elasticity of z for top earners is constant and denoted by $\varepsilon_{z,1-\tau}$.

When tax τ is raised we have no effects on individuals with income below z^*, while all income above z^* are affected by the change. We will study three different effects of the tax.

Mechanical Effect Suppose labor supply was inelastic, when τ increases we would see a mechanical increase in revenues of the following form:

$$dM = d\tau\,(z - z^*)$$

The mechanical effect is proportional to the difference between the average income above z^* and z^*. It measures the mechanical increase in revenues that is generated by the tax change.

Behavioral Effect Top earners react to the tax increase by adjusting their labor supply. The behavioral response triggers a fiscal externality and a reduction in revenues. The behavioral effect is:

$$\begin{aligned} dB &= \tau dz = -\tau \frac{dz}{d\,(1-\tau)} d\tau \\ &= -\frac{\tau}{1-\tau}\frac{1-\tau}{z}\frac{dz}{d\,(1-\tau)} d\tau \\ &= -\frac{\tau}{1-\tau}\varepsilon_{z,1-\tau} z d\tau \end{aligned}$$

It is proportional to the elasticity of labor supply since the more elastic is labor the higher is the revenue loss.

Welfare Effect Denote with $\bar{g}$ the (assumed) constant social marginal welfare weight for earners above z^*. The tax change mechanically raises revenues on top income individuals generating the following welfare effect:

$$dW = d\tau \bar{g}\,(z - z^*)$$

We also showed that the tax increase triggers a behavioral response. The reason it is not included in the welfare effect is that if the tax change is small people reoptimize at the margin and their utility level is unaffected. Again, this is an Envelope theorem argument.

Optimal Tax In equilibrium the three effects must sum to zero. If they did not the government would have margin to adjust the tax rate and achieve a higher social welfare. We therefore have:

$$dM + dB + dW = d\tau \left[(1 - \bar{g}) \left[z - z^* \right] - \varepsilon_{z,1-\tau} \frac{\tau}{1-\tau} z \right] = 0$$

Rearranging:

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + a\varepsilon_{z,1-\tau}}$$

with $a = \frac{z}{z - z^*}$ measuring the thinness of the right tail in the income distribution. The optimal tax is decreasing in the social marginal welfare weight of top earners $\bar{g}$: the more the government cares about top income individuals, the less they will be taxed. As we could expect, the optimal tax is also decreasing in the elasticity of labor supply. Finally, τ^* decreases in a. The shape of the income distribution matters: the government sets lower top income taxes when earners above z^* are mostly concentrated around z^*. If instead there is a thicker tail, the top income tax is higher.