

# Lecture 2: Theoretical Tools for Public Economics

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## THEORETICAL AND EMPIRICAL TOOLS

**Theoretical tools:** The set of tools designed to understand the mechanics behind economic decision making.

Economists model individuals' choices using the concepts of utility function maximization subject to budget constraint

**Empirical tools:** The set of tools designed to analyze data and answer questions raised by theoretical analysis.

## UTILITY MAPPING OF PREFERENCES

**Utility function:** A utility function is some mathematical function translating consumption into utility:

$$U = u(X_1, X_2, X_3, \dots)$$

where  $X_1, X_2, X_3$ , and so on are the quantity of goods 1,2,3,... consumed by the individual

Example with two goods:  $u(X_1, X_2) = \sqrt{X_1 \cdot X_2}$  with  $X_1$  number of movies,  $X_2$  number of music songs

Individual utility increases with the level of consumption of each good

## PREFERENCES AND INDIFFERENCE CURVES

**Indifference curve:** A graphical representation of all bundles of goods that make an individual equally well off

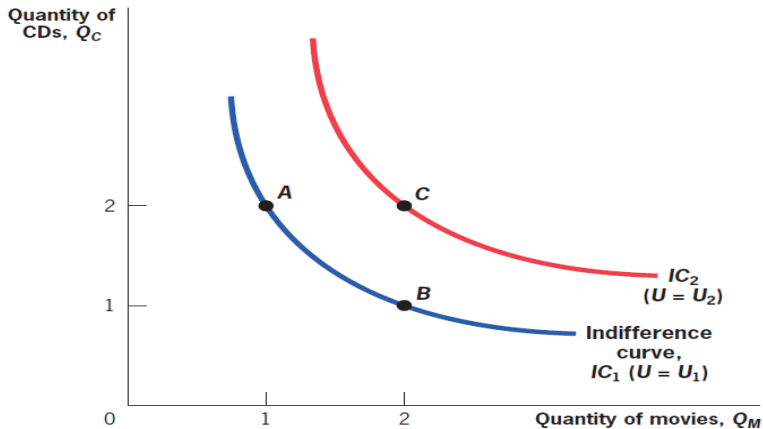
Mathematically, indifference curve giving utility level  $\bar{U}$  is given by the set of bundles  $(X_1, X_2)$  such that  $u(X_1, X_2) = \bar{U}$

Indifference curves have two essential properties, both of which follow naturally from the more-is-better assumption:

1. Consumers prefer higher indifference curves.
2. Indifference curves are always downward sloping.

## 2.1

## Preferences and Indifference Curves



## MARGINAL UTILITY

**Marginal utility:** The additional increment to utility obtained by consuming an additional unit of a good:

Marginal utility of good 1 is defined as:

$$MU_1 = \frac{\partial u}{\partial X_1} \simeq \frac{u(X_1 + dX_1, X_2) - u(X_1, X_2)}{dX_1}$$

It is the derivative of utility with respect to  $X_1$  keeping  $X_2$  constant (called the partial derivative)

Example:

$$u(X_1, X_2) = \sqrt{X_1 \cdot X_2} \Rightarrow \frac{\partial u}{\partial X_1} = \frac{\sqrt{X_2}}{2\sqrt{X_1}}$$

This utility function described exhibits the important principle of **diminishing marginal utility**:  $\partial u / \partial X_1$  decreases with  $X_1$ : the consumption of each additional unit of a good gives less extra utility than the consumption of the previous unit

## MARGINAL RATE OF SUBSTITUTION

**Marginal rate of substitution (MRS):** The *MRS* is equal to (minus) the slope of the indifference curve, the rate at which the consumer will trade the good on the vertical axis for the good on the horizontal axis.

Marginal rate of substitution between good 1 and good 2 is:

$$MRS_{1,2} = \frac{MU_1}{MU_2}$$

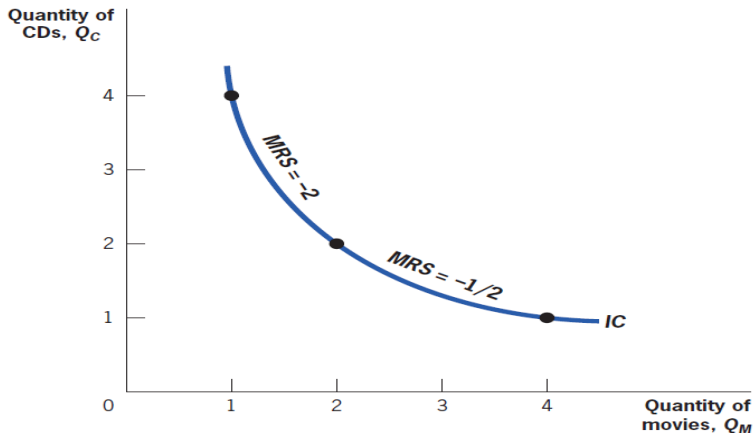
Individual is indifferent between 1 unit of good 1 and  $MRS_{1,2}$  units of good 2.

Example:

$$u(X_1, X_2) = \sqrt{X_1 \cdot X_2} \Rightarrow MRS_{1,2} = \frac{X_2}{X_1}$$

## 2.1

## Marginal Rate of Substitution



## BUDGET CONSTRAINT

**Budget constraint:** A mathematical representation of all the combinations of goods an individual can afford to buy if she spends her entire income.

$$p_1X_1 + p_2X_2 = Y$$

with  $p_i$  price of good  $i$ , and  $Y$  disposable income.

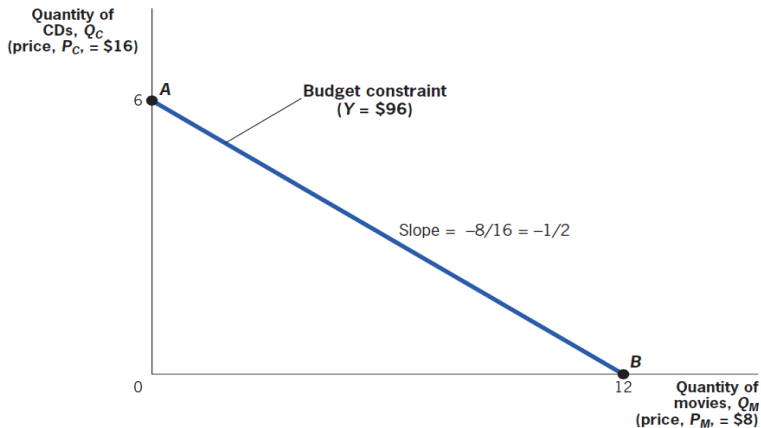
Budget constraint defines a linear set of bundles the consumer can purchase with its disposable income  $Y$

$$X_2 = \frac{Y}{p_2} - \frac{p_1}{p_2}X_1$$

The slope of the budget constraint is  $-p_1/p_2$

## 2.1

## Budget Constraints



## UTILITY MAXIMIZATION

Individual maximizes utility subject to budget constraint:

$$\max_{X_1, X_2} u(X_1, X_2) \quad \text{subject to} \quad p_1 X_1 + p_2 X_2 = Y$$

$$\text{Solution:} \quad MRS_{1,2} = \frac{p_1}{p_2}$$

Proof: Budget implies that  $X_2 = (Y - p_1 X_1) / p_2$

Individual chooses  $X_1$  to maximize  $u(X_1, (Y - p_1 X_1) / p_2)$

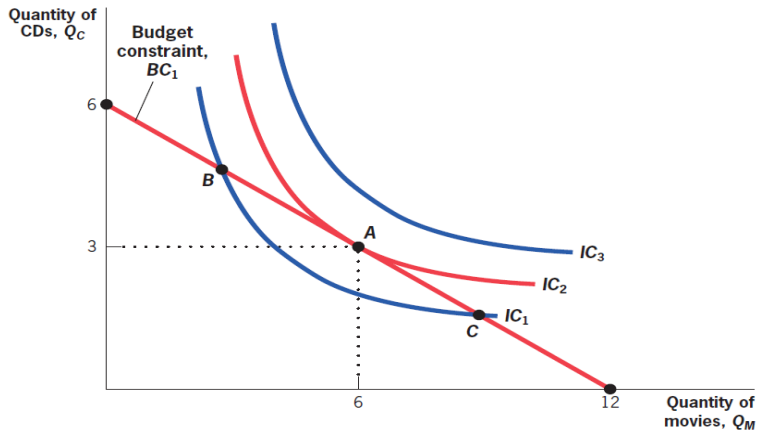
The first order condition (FOC) is:

$$\frac{\partial u}{\partial X_1} - \frac{p_1}{p_2} \cdot \frac{\partial u}{\partial X_2} = 0.$$

At the optimal choice, the individual is indifferent between buying 1 extra unit of good 1 for \$  $p_1$  and buying  $p_1 / p_2$  extra units of good 2 (also for \$  $p_1$ ).

## 2.1

## Putting It All Together: Constrained Choice



## INCOME AND SUBSTITUTION EFFECTS

Let us denote by  $p = (p_1, p_2)$  the price vector

Individual maximization generates demand functions  $X_1(p, Y)$  and  $X_2(p, Y)$

Why does  $X_1$  depend on  $p$  (not only  $p_1$ )? How does  $X_1(p, Y)$  vary with  $p$  and  $Y$ ?

Those are called price and income effects

Example:  $u(X_1, X_2) = \sqrt{X_1 \cdot X_2}$  then  $MRS_{1,2} = X_2/X_1$ .

Utility maximization implies  $X_2/X_1 = p_1/p_2$  and hence  $p_1X_1 = p_2X_2$

Budget constraint  $p_1X_1 + p_2X_2 = Y$  implies  $p_1X_1 = p_2X_2 = Y/2$

Demand functions:  $X_1(p, Y) = Y/(2p_1)$  and  $X_2(p, Y) = Y/(2p_2)$

## INCOME EFFECTS

Income effect is the effect of giving extra income  $Y$  on the demand for goods: How does  $X_1(p, Y)$  vary with  $Y$ ?

**Normal goods:** Goods for which demand increases as income  $Y$  rises:  $X_1(p, Y)$  increases with  $Y$  (most goods are normal)

**Inferior goods:** Goods for which demand falls as income  $Y$  rises:  $X_1(p, Y)$  decreases with  $Y$  (example: you use public transportation less when you are rich enough to buy a car)

Example: if leisure is a normal good, you work less (i.e. get more leisure) if you are given a transfer

## PRICE EFFECTS

How does  $X_1(p_1, p_2, Y)$  vary with  $p_1$ ?

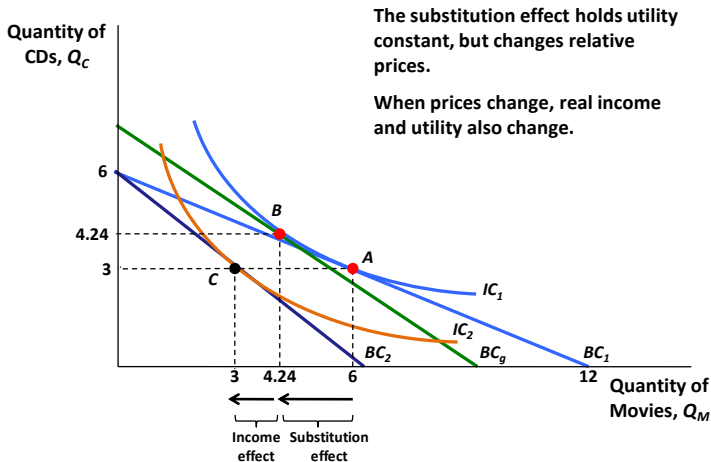
Changing  $p_1$  affects the slope of the budget constraint and can be decomposed into 2 effects:

- 1) **Substitution effect:** Holding utility constant, a relative rise in the price of a good will always cause an individual to choose less of that good
- 2) **Income effect:** A rise in the price of a good will typically cause an individual to choose less of all goods because her income can purchase less than before

For normal goods, an increase in  $p_1$  reduces  $X_1(p_1, p_2, Y)$  through both substitution and income effects

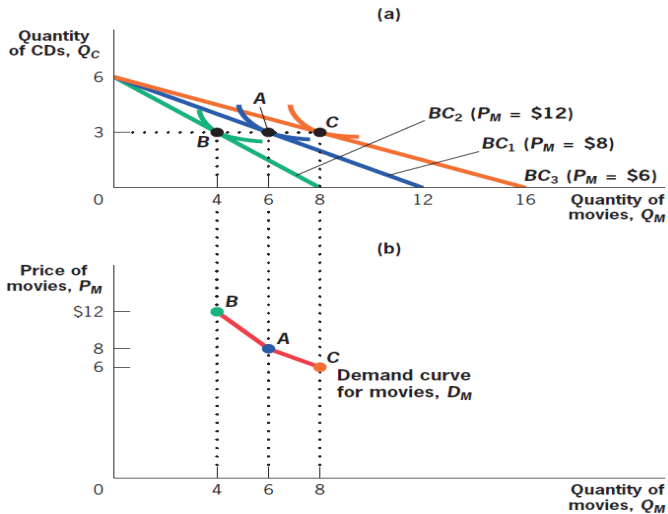
## 2.1

## The Effects of Price Changes: Substitution and Income Effects



## 2.3

## Demand Curves



## ELASTICITY OF DEMAND

Each individual has a demand for each good that depends on the price  $p$  of the good. Aggregating across all individuals, we get aggregate demand  $D(p)$  for the good

At price  $p$ , demand is  $D(p)$  and  $p$  is the \$ value for consumers of the marginal (last) unit consumed

Demand graph: quantity on X-axis, price on Y-axis

**Elasticity of demand**  $\varepsilon$ : The % change in demand caused by a 1% change in the price of that good:

$$\varepsilon = \frac{\% \text{ change in quantity demanded}}{\% \text{ change in price}} = \frac{\Delta D / D}{\Delta p / p} = \frac{p}{D} \frac{dD}{dp}$$

Elasticities are widely used because they are **unit free**

## PROPERTIES OF ELASTICITY OF DEMAND

- 1) Typically negative, since quantity demanded typically falls as price rises.
- 2) Typically not constant along a demand curve.
- 3) With vertical demand curve, demand is **perfectly inelastic** ( $\varepsilon = 0$ ).
- 4) With horizontal demand curve, demand is **perfectly elastic** ( $\varepsilon = -\infty$ ).
- 5) The effect of one good's prices on the demand for another good is the **cross-price** elasticity. Typically, not zero.

## PRODUCERS

Producers (typically firms) use technology to transform inputs (labor and capital) into outputs (consumption goods)

Goal of producers is to maximize profits = sales of outputs minus costs of inputs

Production decisions (for given prices) define supply functions

## SUPPLY CURVES

**Marginal cost:** The incremental cost to a firm of producing one more unit of a good

**Profits:** The difference between a firm's revenues and costs, maximized when marginal revenues equal marginal costs

**Supply curve**  $S(p)$  is the quantity that firms in aggregate are willing to supply at each price: typically upward sloping with price due to decreasing returns to scale

At price  $p$ , producers produce  $S(p)$ , and the \$ cost of producing the marginal (last) unit is  $p$

Elasticity of supply  $\epsilon_S$  is defined as

$$\epsilon_S = \frac{\% \text{ change in quantity supplied}}{\% \text{ change in price}} = \frac{\Delta S/S}{\Delta p/p} = \frac{p}{S} \frac{dS}{dp}$$

## MARKET EQUILIBRIUM

Demanders and suppliers interact on markets

**Market equilibrium:** The equilibrium is the price  $p^*$  such that

$$D(p^*) = S(p^*)$$

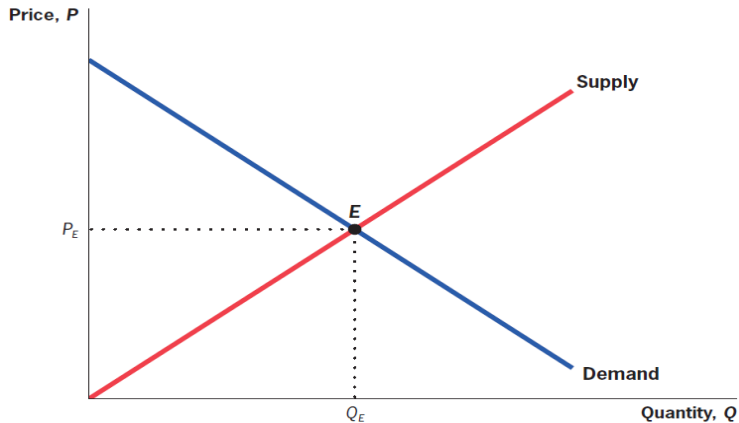
In the simple diagram,  $p^*$  is unique if  $D(p)$  decreases with  $p$  and  $S(p)$  increases with  $p$

If  $p > p^*$ , then supply exceeds demand, and price needs to fall to equilibrate supply and demand

If  $p < p^*$ , then demand exceeds supply, and price needs to increase to equilibrate supply and demand

## 2.3

## Equilibrium: Graphical Representation



## SOCIAL EFFICIENCY

Social efficiency represents the net gains to society from all trades that are made in a particular market, and it consists of two components: consumer and producer surplus.

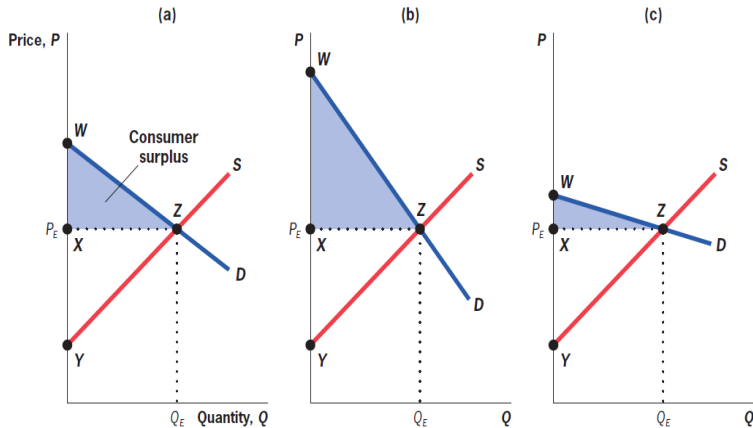
**Consumer surplus:** The benefit that consumers derive from consuming a good, above and beyond the price they paid for the good. It is the area below demand curve and above market price.

**Producer surplus:** The benefit producers derive from selling a good, above and beyond the cost of producing that good. It is the area above supply curve and below market price.

**Total social surplus (social efficiency):** The sum of consumer surplus and producer surplus. It is the area above supply curve and below demand curve.

## 2.3

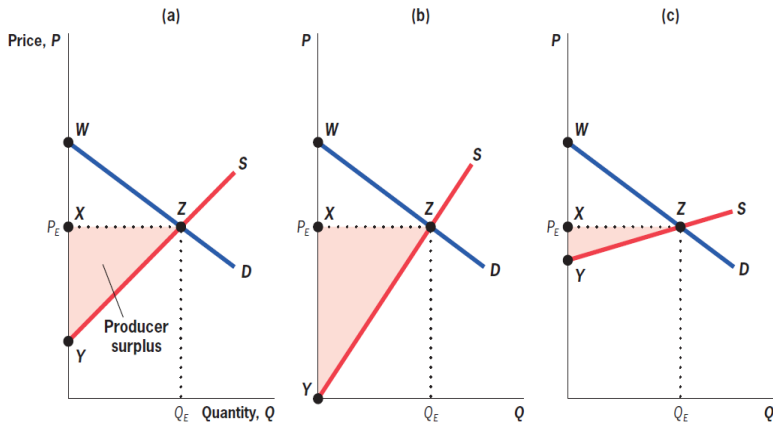
## Consumer Surplus: Graphical Representation



- Consumer surplus is the area under the demand curve since demand = willingness to pay.

## 2.3

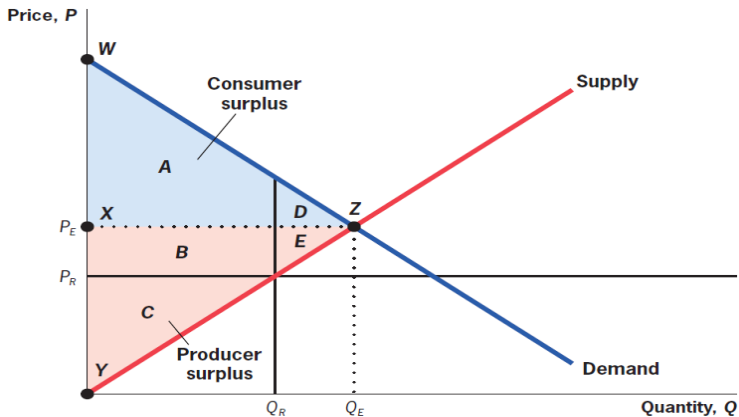
## Producer Surplus: Graphical Representation



- Producer surplus is the area above the supply curve since supply = marginal cost.

## 2.3

## Social Surplus: Graphical Representation



## COMPETITIVE EQUILIBRIUM MAXIMIZES SOCIAL EFFICIENCY

### **First Fundamental Theorem of Welfare Economics:**

The competitive equilibrium where supply equals demand, maximizes social efficiency.

**Deadweight loss:** The reduction in social efficiency from denying trades for which benefits exceed costs when quantity differs from the socially efficient quantity

Key rule: Deadweight loss triangles point to the social optimum, and grow outward from there.

The simple efficiency result from the 1-good diagram can be generalized into the first welfare theorem (Arrow-Debreu, 1940s), most important result in economics

## Generalization: 1st Welfare Theorem

**1st Welfare Theorem:** If (1) no externalities, (2) perfect competition [individuals and firms are price takers], (3) perfect information, (4) agents are rational, then private market equilibrium is **Pareto efficient**

**Pareto efficient:** Impossible to find a technologically feasible allocation that improves everybody's welfare

Pareto efficiency is desirable but a very weak requirement (a single person consuming everything is Pareto efficient)

Government intervention may be particularly desirable if the assumptions of the 1st welfare theorem fail, i.e., when there are market failures  $\Rightarrow$  Govt intervention can potentially improve everybody's welfare

Second part of class considers such market failure situations

## 2nd Welfare Theorem

Even with no market failures, free market outcome might generate substantial inequality. Inequality is seen as the biggest issue with market economies.

**2nd Welfare Theorem:** Any Pareto Efficient allocation can be reached by

(1) Suitable redistribution of initial endowments [individualized **lump-sum** taxes based on individual characteristics and not behavior]

(2) Then letting markets work freely

⇒ No conflict between efficiency and equity

## 2nd Welfare Theorem fallacy

In reality, 2nd welfare theorem does not work because redistribution of initial endowments is not feasible (because initial endowments cannot be observed by the government)

⇒ govt needs to use **distortionary** taxes and transfers based on economic outcomes (such as income or working situation)

⇒ Conflict between efficiency and equity: **Equity-Efficiency trade-off**

First part of class considers policies that trade-off equity and efficiency

## Illustration of 2nd Welfare Theorem Fallacy

Suppose economy is populated 50% with disabled people unable to work (hence they earn \$0) and 50% with able people who can work and earn \$100

**Free market outcome:** disabled have \$0, able have \$100

**2nd welfare theorem:** govt is able to tell apart the disabled from the able [even if the able do not work]

⇒ can tax the able by \$50 [regardless of whether they work or not] to give \$50 to each disabled person ⇒ the able keep working [otherwise they'd have zero income and still have to pay \$50]

**Real world:** govt can't tell apart disabled from non working able

⇒ \$50 tax on workers + \$50 transfer on non workers destroys all incentives to work ⇒ govt can no longer do full redistribution ⇒ Trade-off between equity and size of the pie

## SOCIAL WELFARE FUNCTIONS

Economists evaluate welfare using social welfare functions

**Social welfare function (SWF):** A function that combines the utility functions of all individuals into an overall social utility function.

## UTILITARIAN SOCIAL WELFARE FUNCTION

With a utilitarian social welfare function, society's goal is to maximize the sum of individual utilities:

$$SWF = U_1 + U_2 + \dots + U_N$$

The utilities of all individuals are given equal weight, and summed to get total social welfare

If marginal utility of money decreases with income (satiation), utilitarian criterion values redistribution from rich to poor

Taking \$1 for a rich person decreases his utility by a small amount, giving the \$1 to a poor person increases his utility by a large amount  $\Rightarrow$  Transfers from rich to poor increase total utility

## RAWLSIAN SOCIAL WELFARE FUNCTION

Rawls (1971) proposed that society's goal should be to maximize the well-being of its worst-off member. The Rawlsian SWF has the form:

$$SWF = \min(U_1, U_2, \dots, U_N)$$

Since social welfare is determined by the minimum utility in society, social welfare is maximized by maximizing the well-being of the worst-off person in society (=maxi-min)

Rawlsian criterion is even more redistributive than utilitarian criterion: society wants to extract as much tax revenue as possible from the middle and rich to make transfers to the poor as large as possible

## OTHER SOCIAL JUSTICE PRINCIPLES

Standard welfarist approach is based on individual utilities. This fails to capture important elements of actual debates on redistribution and fairness

**1) Just deserts:** Individuals should receive compensation congruent with their contributions (libertarian)

**2) Commodity egalitarianism:** Society should ensure that individuals meet a set of basic needs (seen as rights) but that beyond that point income distribution is irrelevant

⇒ Rich countries today consider free K-12 education, universal health care, decent retirement/disability benefits as rights

**3) Equality of opportunity:** Society should ensure that all individuals have equal opportunities for success

⇒ Individuals should be compensated for inequalities they are not responsible for (e.g., family background, inheritance, intrinsic ability) but not for inequalities they are responsible for (being hard working vs. loving leisure)

## TESTING PEOPLE SOCIAL PREFERENCES

Saez-Stantcheva '16 survey people online (using Amazon MTurk) by asking hypothetical questions to elicit social preferences. Key findings:

- 1) People typically do not have “utilitarian” social justice principles (consumption lover not seen as more deserving than frugal person)
- 2) People put weight on whether income has been earned through effort vs. not (hard working vs. leisure lover)
- 3) People put a lot of weight of what people would have done absent the government intervention (deserving poor vs. free loaders)

**Which of the following two individuals do you think is most deserving of a \$1,000 tax break?**

Individual A earns \$50,000 per year, pays \$10,000 in taxes and hence nets out \$40,000. She greatly enjoys spending money, going out to expensive restaurants, or traveling to fancy destinations. She always feels that she has too little money to spend.

Individual B earns the same amount, \$50,000 per year, also pays \$10,000 in taxes and hence also nets out \$40,000. However, she is a very frugal person who feels that her current income is sufficient to satisfy her needs.

- 
- Individual A is most deserving of the \$1,000 tax break
  - Individual B is most deserving of the \$1,000 tax break
  - Both individuals are exactly equally deserving of the tax \$1,000 break

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**Which of the following two individuals is most deserving of a \$1,000 tax break?**

Individual A earns \$30,000 per year, by working in two different jobs, 60 hours per week at \$10/hour. She pays \$6,000 in taxes and nets out \$24,000. She is very hard-working but she does not have high-paying jobs so that her wage is low.

Individual B also earns the same amount, \$30,000 per year, by working part-time for 20 hours per week at \$30/hour. She also pays \$6,000 in taxes and hence nets out \$24,000. She has a good wage rate per hour, but she prefers working less and earning less to enjoy other, non-work activities.

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- Individual A is most deserving of the \$1,000 tax break
- Individual B is most deserving of the \$1,000 tax break
- Both individuals are exactly equally deserving of the \$1,000 tax break

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**We assume now that the government can increase benefits by \$1,000 for some recipients of government benefits.**

**Which of the following four individuals is most deserving of the \$1,000 increase in benefits?**

**Please drag and drop the four individuals into the appropriate boxes on the left. The upper box, marked 1 should contain the individual you think is most deserving. The box labeled "2" should contain the second most-deserving individual, etc.. Please note that you can put two individuals in the same box if you think that they are equally deserving.**

Individual A gets \$15,000 per year in Disability Benefits because she cannot work due to a disability and has no other resources.

Individual B gets \$15,000 per year in Unemployment Benefits and has no other resources. She lost her job and has not been able to find a new job even though she has been actively looking for one.

Individual C gets \$15,000 per year in Unemployment Benefits and has no other resources. She lost her job but has not been looking actively for a new job, because she prefers getting less but not having to work.

Individual D gets \$15,000 per year in Welfare Benefits and Food Stamps and has no other resources. She is not looking for a job actively because she can get by living off those government provided benefits.

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Source: survey in Saez and Stantcheva (2013)

**Table 2: Revealed Social Preferences**

	(1)	(2)	(3)	(4)
<b>A. Consumption lover vs. Frugal</b>				
	Consumption lover > Frugal	Consumption lover = Frugal	Consumption lover < Frugal	
# obs. = 1,125	4.1%	74.4%	21.5%	
<b>B. Hardworking vs. leisure lover</b>				
	Hardworking > Leisure lover	Hardworking = Leisure lover	Hardworking < Leisure lover	
# obs. = 1,121	42.7%	54.4%	2.9%	
<b>C. Transfer Recipients and free loaders</b>				
	Disabled person unable to work	Unemployed looking for work	Unemployed not looking for work	Welfare recipient not looking for work
# obs. = 1,098				
Average rank (1-4) assigned	1.4	1.6	3.0	3.5
% assigned first rank	57.5%	37.3%	2.7%	2.5%
% assigned last rank	2.3%	2.9%	25.0%	70.8%

Notes: This table reports preferences for giving a tax break and or a benefit increase across individuals in various scenarios. Panel A considers two individuals with the same earnings, same taxes, and same disposable income but high marginal utility of income (consumption lover) vs. low marginal utility of income (frugal). In contrast to utilitarianism, 74% of people report that consumption loving is irrelevant and 21.5% think the frugal person is most deserving. Panel B considers two individuals with the same earnings, same taxes, and same disposable income but different wage rates and hence different work hours. 54.4% think hours of work is irrelevant and 42.7% think the

## ACTUAL SOCIAL PREFERENCES

**General conclusion:** People favor redistribution if they feel inequalities are “unfair” but unfair means different things to different people

⇒ Redistribution supported when people don't have control [education for children, health insurance for the sick, retirement/disability benefits for the elderly/disabled unable to work]

⇒ Less support when people have some or full control [unemployment, being low income]

Conservatives tend to frame things: individuals have control (personal responsibility), govt should just enforce rules

Liberals tend to frame things: many forces in society beyond individuals' control (“we are all in this together”), society should provide nurturing

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