

Practice Exam

EC2450A

Fall 2016

Shorter Questions

For these questions write out detailed explanations since all points will be given based on the explanation. Write out formulas and equations if needed to explain your answers. If the answer is ambiguous, you need to explain the different cases that could arise and the assumptions under which a statement may or may not be true.

- 1) Goods which are consumed by high-income agents, such as luxury cars, should be taxed more. Comment.
- 2) Should the tax rate on capital income be much lower than the tax rate on labor income, since capital is much more elastic than labor?
- 3) Optimal income tax formulas are not affected by whether top incomes are mostly coming from productive effort or from rent-seeking effort. True or False?

Problem 1

Consider agents with utility functions $u_i(c_i, z_i)$ where i indexes the agent, c_i is consumption and z_i is income earned.

The social welfare function is

$$SWF = \int_i g_i u_i(c_i, z_i) di$$

- i) Write down an agent's budget constraint if there is a linear tax on income τ , the revenues from which are rebated lump-sum to all agents.
- ii) Write down the maximization problem of the government with a linear tax and derive the optimal linear tax rate τ^* .
- iii) Explain the formula you obtain. Give an estimate of what the elasticity could be, based on work you have read. What is the linear optimal Rawlsian tax rate?
- iv) Suppose now that the government has a nonlinear tax available $T(z)$. Write down an agent's budget constraint under the nonlinear tax.
- v) Let us assume that utility is quasilinear. $u_i(c_i, z_i) = c_i - h_i(z_i)$. Set up the government's maximization problem and derive the optimal nonlinear tax using a perturbation argument.

vi) Explain what the optimal tax rate depends on. Give an example of social welfare weights g_i that would generate a particular form of the tax formula.

vii) Explain when the top income tax rate would be zero. Do you think this result makes empirical sense?

Problem 2

Individuals have utility: $c - v(l)$, where c is consumption and l is labor supply. Individuals have a skill n that is exogenously given and equal to marginal productivity. Earnings are $z = nl$. The density of skills is $f(n)$ and the CDF is $F(n)$ on $[0, \infty)$.

The government does not observe the skill. Hence, the tax is based on income z , $T(z)$.

i) Write down the incentive constraint(s) for each type n . How many such constraints are there for each type n ?

ii) Show that the incentive constraints imply a first-order condition for each type n and that this first-order condition can be replaced in turn by an envelope condition. Are the problems with the incentive constraints from part i) and the problem with the envelope condition from part ii) equivalent? Why or why not? Are there special conditions under which your answer would differ?

iii) Express the elasticity of labor supply $e = \frac{dl}{d(n(1-\tau))} \frac{(1-\tau)n}{l}$ as a function of the utility parameters and labor supply. Is this a compensated or an uncompensated elasticity?

iv) Write down the resource constraint if there is an exogenous revenue requirement (non-transfer related) E .

v) Suppose that the government maximizes a social welfare function

$$SWF = \int_n G(u_n) f(n) dn$$

Set up the government's maximization problem with the appropriate constraints.

vi) Explain how you will solve this problem using optimal control methods. What are your choice and your state variables? Write down the Hamiltonian.

vii) Take the first-order conditions and rearrange them. Give an expression for the multiplier on the budget constraint as a function of the social welfare weights, using the transversality conditions on the co-states. Explain what this expression says.

viii) In the first-order condition for labor/income, make the marginal tax rate appear using the first-order condition of the agent if the agent faced the nonlinear tax schedule $T(z)$.

ix) Use the expression for e that you found earlier to make e appear in the FOC for l or z . Rearrange to obtain the familiar marginal tax formula.

x) Give an expression that links the density of types $f(n)$ and the density of income $h(z)$.

xi) Use the expression in x) to rewrite the tax formula in terms of observables (incomes z and their distributions) rather than in terms of unobservables (types n and their distributions).