Economics 2450A: Public Economics
Section 6: Optimal Income Transfers

Matteo Paradisi

October 17, 2016

In this Section we study the optimal design of income transfers. We start from a formal model where we specify individuals' preferences and a government's social welfare function. We then take the approach by Saez (2002) to derive optimal transfers using an "experiment" where we introduce a perturbation around the optimal tax schedule for a generic "occupation" and derive a formula for the optimal tax.

1 Optimal Income Transfers in a Formal Model

We introduce in this paragraph a model of discrete choices where we will derive optimal taxes. Suppose agents choose an occupation i among a set of occupations $\{1, 2, \ldots, I\}$ and earn income w_i at occupation i. Each individual is indexed by $m \in M$ being a multidimensional set of measure one. The measure of individuals on M is denoted by $dv(m)$. The agents maximize $u^m(c_i, i)$ differentiable in consumption. Individual consumption after taxes is $c_i = w_i - T_i$. A tax schedule defines a vector $(c_0, \ldots, c_I)$ such that the set M will be partitioned in subsets $M_1, M_2, \ldots, M_I$. Denote with $h_i(c_0, c_1, \ldots, c_I)$ the fraction of individuals choosing occupation i such that $\sum_i h_i = 1$. h_i is differentiable under the assumption that tastes for work captured by $u^m(\cdot)$ are regularly distributed. We define the elasticity of participation for occupation i as follows:

$$\eta_i = \frac{c_i - c_0}{h_i} \frac{\partial h_i}{\partial (c_i - c_0)} \tag{1}$$

Suppose the government weights individual utilities through linear welfare weights μ^m and that the social welfare function is:

$$W = \int_M \mu^m u^m (w_{i^*} - T_{i^*}, i^*) \, dv(m) \tag{2}$$

The government has some revenue requirement H such that the budget can be written as:

$$\sum_i h_i T_i = H \tag{3}$$

We solve the problem with a Lagrangian where we attach multiplier λ to the government constraint. The FOC wrt T_i reads:

$$-\int_{M_i} \mu^m \frac{\partial u^m (c_{i^*}, i^*)}{\partial c_i} dv(m) + \lambda \left[h_i - \sum_{j=0}^{I} T_j \frac{\partial h_j}{\partial c_i} \right] = 0 \tag{4}$$

For the usual envelope argument equation (4) ignores the welfare effect of a change in c_i.

A social marginal welfare weight is:

$$g_i = \frac{1}{\lambda h_i} \int_{M_i} \mu^m \frac{\partial u^m (c_{i^*}, i^*)}{\partial c_i} dv(m) \tag{5}$$

Using the definition of g_i we can rewrite (4) as:

$$(1-g_i)\,h_i = \sum_{j=0}^{I} T_j \frac{\partial h_j}{\partial c_i} \tag{6}$$

This formula is very similar to the one you will see in the spring studying Ramsey taxation.[1] Take a benchmark case of no income effects such that $h_j(c_0,\dots,c_I) = h_j(c_0+R,\dots,c_I+R)$, the formula implies that $(1-g_i)\,h_i = 0$. Summing over all is:

$$\sum_i h_i g_i = \sum_i h_i = 1 \tag{7}$$

2 Optimal Tax/Transfer with Extensive Margin Only

Suppose each individual only chooses between some occupation i and being unemployed. This can be rationalized by a utility function where $u^m(c_j, j) = -\infty$ for any $j \neq i$. The assumption implies that $\partial h_i/\partial c_i + \partial h_0/\partial c_i = 0$ and we can rewrite (6) as:

$$(1-g_i)\,h_i = T_i \frac{\partial h_i}{\partial c_i} + T_0 \frac{\partial h_0}{\partial c_0} = (T_i - T_0)\frac{\partial h_i}{\partial c_i}$$

using the definition of the elasticity of participation:

$$\frac{T_i - T_0}{c_i - c_0} = \frac{1}{\eta_i}(1-g_i) \tag{8}$$

The level of taxation at occupation i decreases in the elasticity of taxation for the usual efficiency argument.

Redistributive preferences imply $g_0 \geq g_1 \geq \ldots \geq g_I$. Suppose there are no income effects, we know from (7) that the weighted average of the g_is is 1 and therefore there is a i^* such that $g_j \leq 1$ for $j \leq i^*$ and $g_j > 1$ for $j > i^*$. This implies that $T_i - T_0 \leq 0$ for $i \leq i^*$, meaning that the government is providing a higher transfer to workers with low income relative to unemployed. Therefore, we established that it is optimal for the government to implement negative marginal tax rates at the bottom of the income distribution.

If the government was Rawlsian, we could have that g_0 only is higher than 1. When this is the case, the tax schedule does not display negative marginal tax rates and we have a classical negative income tax. On the other hand, a utilitarian government would have constant g_is such that the budget constraint is satisfied. We therefore have two cases. First, if every individual can pay H, the government will charge a constant lump-sum tax equal to H to every taxpayer and $g_i = 1$ for every i. Second, if low incomes cannot afford the tax the government will only impose the tax on higher income setting their social marginal welfare weights below 1 and having positive marginal tax rates throughout occupations.

Tax Experiment The same formula for optimal taxes can be derived through the following experiment. Suppose taxes increase by dT_i for occupation i. The mechanical increase in tax revenues is $h_i dT_i$ and it will be valued $(1-g_i)\,h_i dT_i$ by the government taking into account the welfare effect of the change. The government must also account for the fiscal externality generated by the behavioral response of agents in occupation i. Using the elasticity of participation, the share of people leaving occupation i is:

[1] The formula implies that the following is true for every i:

$$\sum_{j=0}^{I} \frac{T_j}{h_i(1-g_i)} \frac{\partial h_j}{\partial c_i} = 1$$

We can interpret the lhs as an index of how much labor supply is discouraged. The formula holds for every i and implies that discouragement is equalized across all occupations.

$$dh_i = -\eta_i \frac{h_i}{c_i - c_0} dT_i$$

Each worker leaving occupation i generates a loss in revenues equal to $T_i - T_0$. The total behavioral effect of the tax increase is:

$$dh_i \left(T_i - T_0\right) = -\eta_i h_i \frac{T_i - T_0}{c_i - c_0} dT_i$$

Summing the mechanical and behavioral effects at the optimum we get:

$$\left(1 - g_i\right) h_i dT_i - \eta_i h_i \frac{T_i - T_0}{c_i - c_0} dT_i = 0$$

Rearranging we can derive (8). The decomposition of the formula in mechanical and behavioral effects provides further intuition for why marginal tax rates can be negative at the optimum. For very low incomes the mechanical effect of providing an extra dollar is positive ($g_i > 1$) and at the same time a decrease in taxes at i provides incentives for unemployed workers to enter the labor force. The sum of the two effects is unambiguously positive.

3 Optimal Tax/Transfer with Intensive Margin Responses

Suppose that agents' preferences are such that they can only work in two adjacent occupations and that we can write the share of workers in occupation i as $h_i\left(c_{i+1} - c_i, c_i - c_{i-1}\right)$ when we assume there are no income effects.[2] The behavioral elasticity is defined as follows:

$$\zeta_i = \frac{c_i - c_{i-1}}{h_i} \frac{\partial h_i}{\partial\left(c_i - c_{i-1}\right)} \tag{9}$$

Equation (6) becomes:

$$\left(1 - g_i\right) h_i = -T_{i+1} \frac{\partial h_{i+1}}{\partial\left(c_{i+1} - c_i\right)} - T_i \frac{\partial h_i}{\partial\left(c_{i+1} - c_i\right)} + T_i \frac{\partial h_i}{\partial\left(c_i - c_{i-1}\right)} + T_{i-1} \frac{\partial h_{i-1}}{\partial\left(c_i - c_{i-1}\right)}$$

By assumption on agent's preferences $\partial h_{i+1}/\partial\left(c_{i+1} - c_i\right) = -\partial h_i/\partial\left(c_{i+1} - c_i\right)$ and rearranging we find:

$$\left(1 - g_i\right) h_i = -\left(T_{i+1} - T_i\right) \frac{\partial h_{i+1}}{\partial\left(c_{i+1} - c_i\right)} + \left(T_i - T_{i-1}\right) \frac{\partial h_i}{\partial\left(c_i - c_{i-1}\right)}$$

Summing over $i, i+1, \ldots, I$ and using the definition in (9) we can derive the optimal tax formula:

$$\frac{T_i - T_{i-1}}{c_i - c_{i-1}} = \frac{1}{\zeta_i} \left[\frac{\left(1 - g_i\right) h_i + \left(1 - g_{i+1}\right) h_{i+1} + \ldots + \left(1 - g_I\right) h_I}{h_i}\right] \tag{10}$$

Non-increasing social marginal welfare weights imply that $\left(1 - g_i\right) h_i + \left(1 - g_{i+1}\right) h_{i+1} + \ldots + \left(1 - g_I\right) h_I \geq 0$ for any $i > 0$. Thus, the tax T_i is increasing in i and it is not optimal to set negative marginal tax rates. Using (7), (10) and computing the formula for the tax rate at the bottom of the income distribution we get:

$$\frac{T_1 - T_0}{c_1 - c_0} = \frac{1}{\zeta_1} \left[\frac{\left(g_0 - 1\right) h_0}{h_1}\right] \tag{11}$$

[2] We can write the share of people working in occupation i as $h_i\left(c_i, c_{i+1}\right)$. No income effects imply $h\left(c_0, c_1, \ldots, c_I\right) = h\left(c_0 + R, c_1 + R, \ldots, c_I + R\right)$. It follows that $h\left(c_i, c_{i+1}\right) = h\left(c_i + c_i - c_{i-1}, c_i + c_{i+1} - c_i\right) = h\left(c_i - c_{i-1}, c_{i+1} - c_i\right)$.

A higher social marginal welfare weight g_0 implies a higher tax rate at the bottom. The reason is that if the government cares more about the unemployed individual it should set the lump-sum transfer $-T_0$ as large as possible by imposing large phasing-out tax rates at the bottom. Negative marginal tax rates at the bottom can still occur for $g_0 < 1$, but this would imply that the unemployed worker has a lower welfare weight than the average taxpayer in the economy, meaning that the government has unusual redistributive tastes.

Tax Experiment The formula in (10) can be derived through an experiment where taxes increase by dT for any occupation $i, i+1, \ldots, i+I$. This change decreases $c_i - c_{i-1}$ by dT and leaves any other difference unaltered. The mechanical increase in revenues is $[h_i + h_{i+1} + \ldots + h_I]\, dT$ and net-of-welfare it is valued $[h_i(1-g_i) + h_{i+1}(1-g_{i+1}) + \ldots + h_I(1-g_I)]\, dT$. The behavioral effect of the tax change arise from individuals in occupation i only when we assume income effects away. The impact on revenues is $dh_i = -h_i \zeta_i dT/(c_i - c_{i-1})$ and it must be scaled by the loss in revenues $T_i - T_{i-1}$ generated by each worker switching to occupation $i-1$. Summing the two impacts:

$$[h_i(1-g_i) + h_{i+1}(1-g_{i+1}) + \ldots + h_I(1-g_I)]\, dT - h_i\zeta_i (T_i - T_{i-1})\, dT/(c_i - c_{i-1}) = 0$$

Rearranging we get the formula in (10). The mechanical and behavioral effects help providing intuition for why negative marginal tax rates are not optimal with intensive margin only. Suppose the government raised taxes at i when there is a negative marginal tax rate in the interval $[i-1, i]$. Individuals would respond by shifting their labor supply to $i-1$ and, given the higher tax rate, would pay more taxes. At the same time the tax change would mechanically increase revenues. Therefore, the government could always improve welfare by increasing taxes as long as the marginal tax rate is negative.

4 Optimal Tax/Transfer with Intensive and Extensive Margin Responses

We present for the sake of simplicity only the tax experiment derivation of the formula. Suppose taxes are raised by dT for everyone in occupation $i, i+1, \ldots, i_I$. The mechanical effect is the same as the one observed in the previous paragraph. However, we have to add the participation effect of an increase in the tax for all the occupations above i. The share of people who become unemployed leaving a generic occupation i is $-h_i\eta_i dT/(c_i - c_0)$, generating a revenue loss equal to $-h_i\eta_i (T_i - T_0)\, dT/(c_i - c_0)$. Summing this effect over every occupation $j \geq i$ and setting the sum of behavioral and mechanical effects equal to 0, we can derive the following formula:

$$\frac{T_i - T_{i-1}}{c_i - c_{i-1}} = \frac{1}{\zeta_i h_i} \sum_{j=i}^{I} h_j \left[1 - g_j - \eta_j \frac{T_j - T_0}{c_j - c_0}\right] \tag{12}$$

When a tax is lowered in the pure extensive margin model, labor supply unambiguously increases. On the other hand, if a tax is decreased in a pure intensive margin model individuals will have incentives to lower their labor supply. The formula shows how to optimally trade-off the two effects.

Notice that (12) can be rewritten as (10) where we employ augmented social welfare weights $\hat{g}_i = g_i + \eta_i (T_i - T_0)/(c_j - c_0)$. When the participation elasticity is high enough, the augmented welfare weights are not necessarily decreasing in w_i if g_is are. This explains why an earning income tax credit could be optimal in a mixed model.

References

[1] "Optimal Income Transfer Programs:Intensive Versus Extensive Labor Supply Responses" Quarterly Journal of Economics, 117, 2002, 1039-1073

[2] "The Case for a Progressive Tax: From Basic Research to Policy Recommendations" with Peter Diamond, Journal of Economic Perspectives 25(4), Fall 2011, 165-190