Optimal Taxation and R&D Policies

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Motivation I: Widespread and Diverse R&D Policies

"The need to foster greater innovation and productivity growth is one of the most important economic challenges we face, and tax policy is one of several important levers that policymakers can use", J. Furman, former chairman of CEA

Businesses spend a lot of resources on R&D... and the government already intervenes heavily.

Large **variety** of policies target **innovation and R&D**

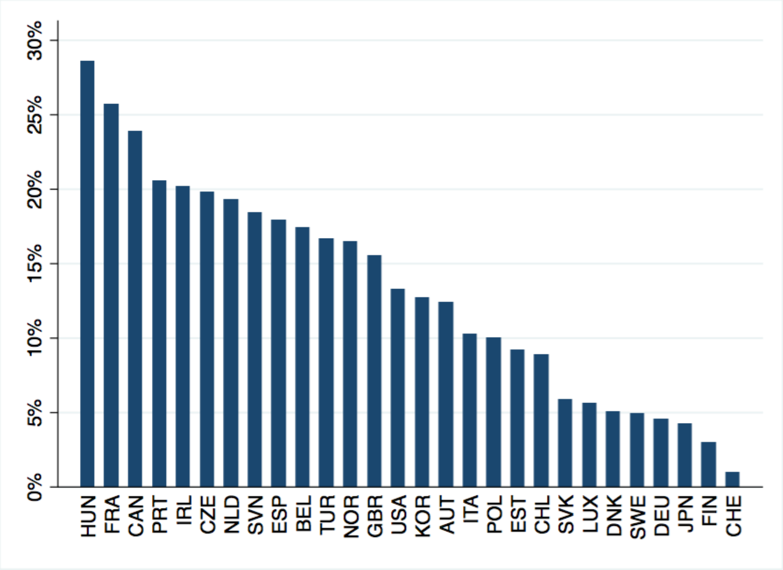
Tax credits, deductions, grants, contracts, direct funding in FFRDCs, Universities, Firms, small business, start-ups..

Large variety of policies across countries as well.

R&D policies are widespread, not fully understood, & very costly:

- "Intramural" R&D cost $35 billion (2014).
- "Extramural" R&D: tax credit $11 bil in 2012, contracting with non FFRDCs 50,6 billion, NSF-NIH $40 billion (econ grant: 0.0025%)

Share of Government Funding in Business R&D



Is the amount spent by government correlated with better productivity?

Motivation II: Private Information is an Important Constraint

- Take young firms at start of their lifecycle. How much of the variation in subsequent innovation quantity & quality can we explain based on observables?
    - Observables: age, assets, past investments, sales, state FE, year FE, sector FE (+ all interactions), and even past innovations:
    - R^2 not above 0.3, improves with age (as info revealed).
    - Conditional on these observables, many "outlier" firms.
- Two ways of possibly addressing asymmetric info problem:
    - **Direct screening**: what the NSF and VCs try to do. Done by the government with public procurement. Hard to do and very costly on a large scale.
    - **Indirect screening**: Design a menu of options (implemented by taxes and subsidies), let firms self-select! "Easy" to decentralize and scalable.

This Paper: Optimal Design of R&D Policy and Firm Taxation

- Firms have **heterogeneous**, stochastic productivities.

  Productivity: efficiency of converting R&D inputs into innovation output.

- **Uncertainty** about R&D returns.
- **Spillovers** between firms: one firm's innovations affect other firms.
- Innovation not appropriable unless IPR.
- Firm productivity is **private information**.
- 1) Mechanism design: no a priori restriction on policy tools.

  Characterize constrained efficient allocations.

  Implementation.

- 2) Quantitative Investigation using Patent data + Compustat data.
- 3) Losses from "simpler" policies (e.g.: linear age-dependent...).

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Related Literature

R&D Policies and Growth: Leahy and Neary (1997), Acemoglu, Akcigit, Bloom and Kerr (2013), Akcigit, Hanley and Serrano-Velarde (2013), Atkeson and Burstein (2014).

Optimal Policy Design: Laffont and Tirole (1986), Sappington (1982), Golosov, Tsyvinski, and Kocherlakota (2003), Kocherlakota (2005), Pavan, Segal and Toikka (2013), Doepke and Townsend (2006), Stantcheva (2012, 2014), Golosov, Tsyvinski and Werning (2006), Farhi and Werning (2013, 2014), Makris and Pavan (2018), Golosov, Troshkin, and Tsyvinski (2016), Troshkin (2018), Ales, Kurnaz, and Sleet (2015), Ales and Sleet (2016),

Heterogeneity in Management: Bloom et al. (2013), Bloom, Sadun and Van Reenen (2012).

Empirics of R&D Policies: Bloom, Griffith, Van Reenen (2002), Hall and Van Reenen (2000), Bloom, Schankerman and Van Reenen (2013).

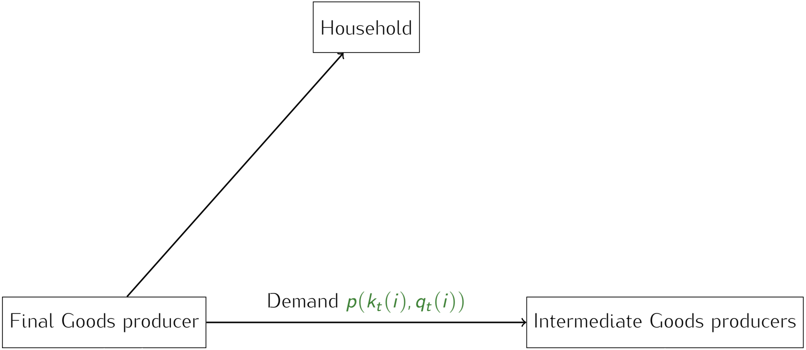
IPR Design: Scotchmer (1999), Kremer (1998), Chari, Golosov and Tsyvinski (2012).

Outline

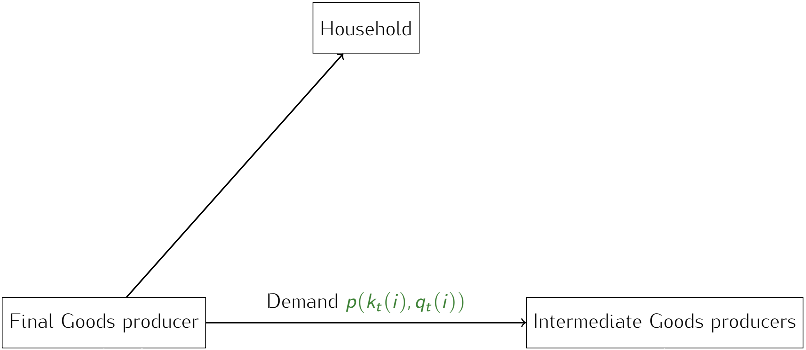
1. Model
2. Optimal Unrestricted Mechanism
3. Quantitative Investigation
4. Optimal Simpler Policies

Model

Intermediate Goods producers



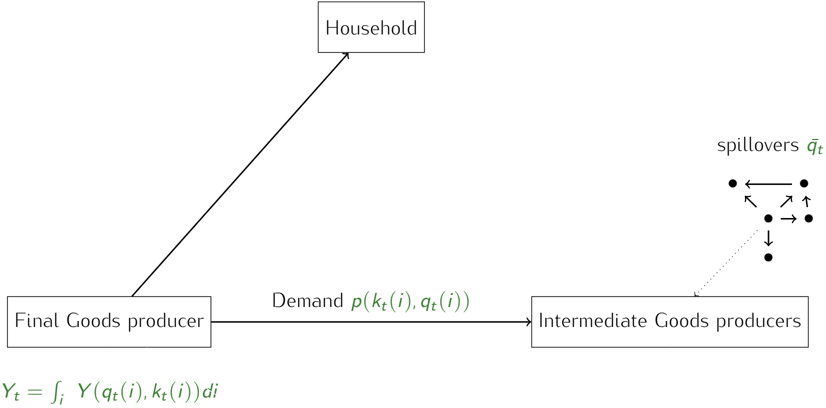
$$Y_t = \int_i Y(q_t(i), k_t(i)) di$$



$Y_t = \int_i \, Y(q_t(i), k_t(i))di$

Production

- Quality $q_t(i)$, quantity $k_t(i)$
- Demand: $p(k_t(i), q_t(i))$



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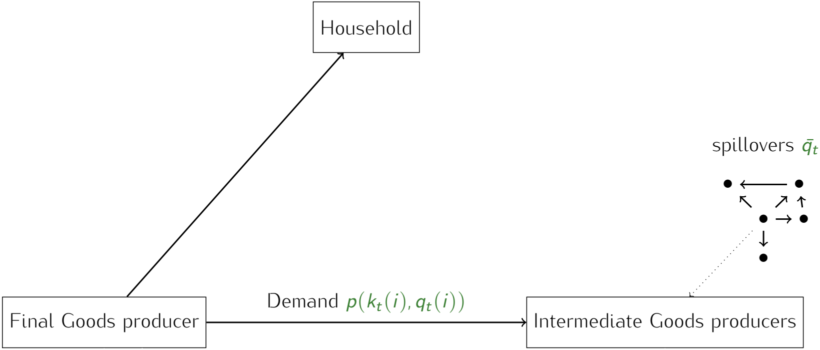

$$Y_t = \int_i \; Y(q_t(i), k_t(i))di$$

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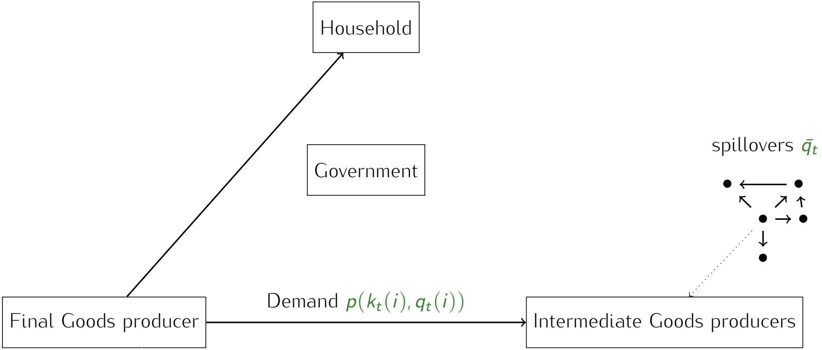
- $\pi(q_t(i), \bar{q}_t) = \max_k \{p(k, q_t(i))k - C(k, \bar{q}_t)\}$



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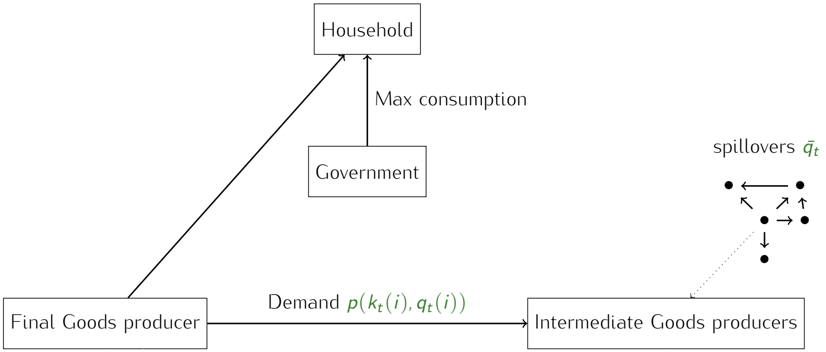

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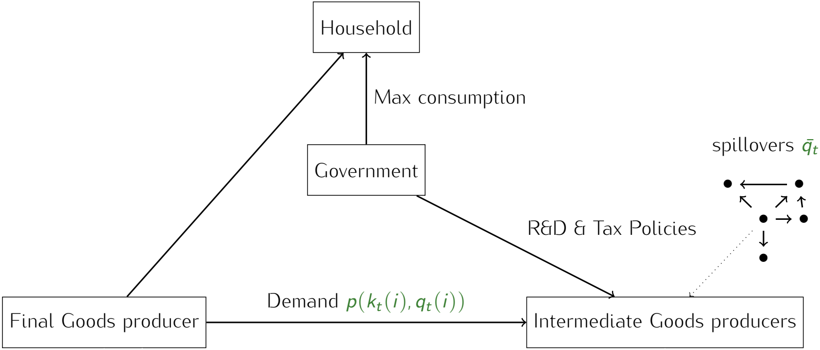

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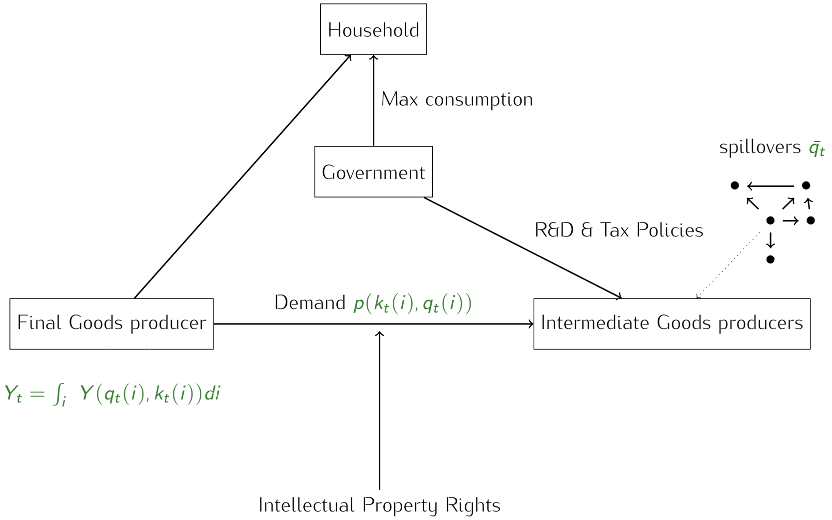
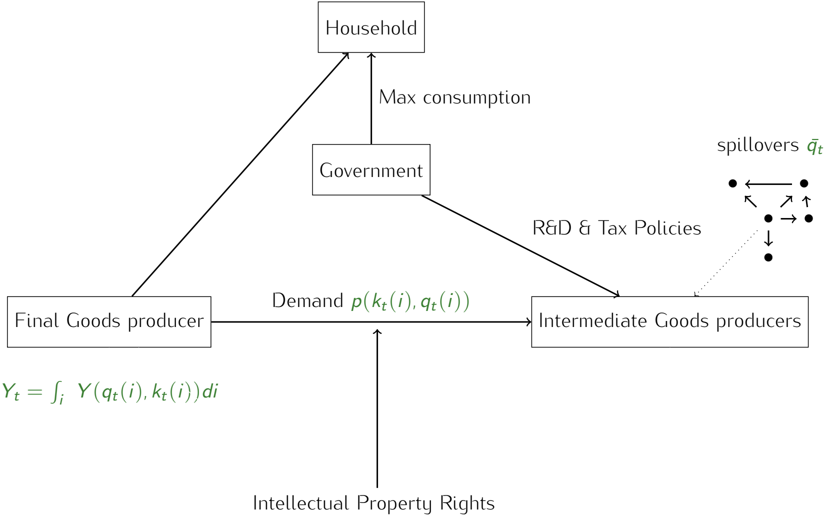

Household
Max consumption
Government
spillovers $\bar{q}_t$
R&D & Tax Policies
Demand $p(k_t(i), q_t(i))$
Final Goods producer
Intermediate Goods producers
$Y_t = \int_i Y(q_t(i), k_t(i))di$
Intellectual Property Rights



- Patent: $p(k_t(i), q_t(i)) = \frac{\partial Y(q_t(i), k_t(i))}{\partial k_t(i)}$
- "Prize": $p(k_t(i), q_t(i)) = \frac{Y(q_t(i), k_t(i))}{k_t(i)}$

Intermediate Producers' static production decisions

- Intermediate good producers: quality $q_t(i)$, quantity $k_t(i)$.
- Final good producer aggregates intermediate goods for consumption:

$$Y_t = \int_i Y(q_t(i), k_t(i))di$$

- Return to quality and quantity for good i: $p(k_t(i), q_t(i))$, depends on intellectual property rights policy.

  Monopoly price: $p(k_t(i), q_t(i)) = \frac{\partial Y(q_t(i), k_t(i))}{\partial k_t(i)}$.

  "Prize" mechanism: $p(k_t(i), q_t(i)) = \frac{Y(q_t(i), k_t(i))}{k_t(i)}$

- Technology spillovers: come from aggregate quality: $\bar{q}_t = \int_i q_t(i)di$
- Cost of production: $C(k_t, \bar{q}_t)$ ($\uparrow$ or $\downarrow$ in $\bar{q}_t$).
- Profit maximization: $\pi(q_t, \bar{q}_t) \equiv \max_k \{p(k, q_t)k - C(k, \bar{q}_t)\}$

Intermediate Producers' Innovation Decisions

- Firms can improve their product quality q_t through R&D and effort: $q_t = H(q_{t-1}, \lambda_t)$.
- The step size $\lambda_t(r_{t-1}, l_t, \theta_t)$ depends on:

  R&D investment r_t cost $M_t(r_t)$.

  R&D effort (unobservable R&D input): l_t at cost $\phi_t(l_t)$.

  Productivity θ_t (managerial/firm quality), Markov $f^t(\theta_t|\theta_{t-1})$, history θ^t.

- $\frac{\partial \lambda}{\partial \theta} > 0$, $\frac{\partial \lambda}{\partial r} > 0$, $\frac{\partial \lambda}{\partial l} > 0$, $\frac{\partial^2 \lambda}{\partial \theta \partial l} > 0$ (screening).
- Returns to R&D are stochastic, depend on stochastic type.

Market Failures and First Best Allocation

1) Lack of appropriability of innovation (need intellectual property rights (IPR)).

2) Technology spillovers.

First best quantity conditional on quality: $k^*(q_t(\theta^t), \bar{q}_t)$.

First best output net of production costs:
$\tilde{Y}^*(q_t(\theta^t), \bar{q}_t) = Y(q_t(\theta^t), k^*(q_t(\theta^t), \bar{q}_t)) - C(k^*(q_t(\theta^t), \bar{q}_t), \bar{q}_t)$.

Optimality: marginal cost = marginal social benefit

$$M_t'(r_t(\theta^t)) = \frac{1}{R}\mathbb{E}\left(\sum_{s=t+1}^{\infty}\left(\frac{1-\delta}{R}\right)^{s-t-1}\left(\frac{\partial\tilde{Y}^*(q_t(\theta^s), \bar{q}_t)}{\partial q_s} + \frac{\partial\tilde{Y}^*(q_t(\theta^s), \bar{q}_t)}{\partial \bar{q}_s}\right)\frac{\partial\lambda_{t+1}}{\partial r_t}\right)$$

$$\phi_t'(l_t(\theta^t)) = \mathbb{E}\left(\sum_{s=t}^{\infty}\left(\frac{1-\delta}{R}\right)^{s-t}\left(\frac{\partial\tilde{Y}^*(q_t(\theta^s), \bar{q}_t)}{\partial q_s} + \frac{\partial\tilde{Y}^*(q_t(\theta^s), \bar{q}_t)}{\partial \bar{q}_s}\right)\right)\frac{\partial\lambda_t(\theta^t)}{\partial l_t(\theta^t)}$$

Asymmetric Information

- Consider two cases.

  (1) Firm productivity θ_t and R&D effort not observable...

  (2) ... and quantity k_t not observable.

- Case (1) $\Leftrightarrow$ can optimize on intellectual property rights policy.

  Optimal IPR trivial here: prize system or patent system + price subsidy.

- Case (2) $\Leftrightarrow$ take IPR as given (partial optimum), e.g.: patents.

- Asymmetric info problem:

  If heterogeneous, but observable types: heterogeneous policies, type-specific lump-sum tax.

  Asymmetric info: cannot extract surplus lump-sum.

  Problem if limited liability and revenue requirement (could be ≤ 0).

Coefficient of Complementarity

- Hicksian coefficient of complementarity between x and y:

$$\rho_{xy} = \frac{\frac{\partial^2 \lambda}{\partial x \partial y}\lambda}{\frac{\partial \lambda}{\partial x}\frac{\partial \lambda}{\partial y}}$$

- $\lambda_t(r, I, \theta) = rI\theta \rightarrow \rho_{\theta I} = \rho_{\theta r} = \rho_{Ir} = 1$.
- $\lambda_t(r, I, \theta) = r + I + \theta \rightarrow \rho_{\theta I} = \rho_{\theta r} = \rho_{Ir} = 0$.
- When $\rho_{\theta r}$ is large: Higher R&D investments increase informational rent that needs to be forfeited to high quality firms.

Comments: Competition and Patent Policy

- Additional observable heterogeneity (sector? product type?) can be conditioned on.
- Competition: exogenous markups.
  - Captured reduced form by (i) cost functions (input market competition) and (ii) substitutability between goods, affects pricing power.
  - Can do comparative statics on R&D policies with respect to competition.
- Different types of innovations: new vs existing product, process vs. product. Common core we focus on: **spillovers**.
- Intellectual Protection Policy: different from R&D policy, but affects it.

Optimal Unrestricted Mechanism

A Direct Revelation Mechanism with Spillovers

Firm reports $\theta'_t(\theta^t)$. History of reports: $\theta'^t = \{\theta'_1(\theta_1), ...\theta'_t(\theta^t)\}$.

Allocations for history of reports: $\{\lambda(\theta'^t),\ r(\theta'^t), T_t(\theta'^t)\}$ (possibly, $k(\theta'^t)$).

Maximize household consumption:

$$\mathbb{E}\left\{\sum_{t=1}^{T}\left(\frac{1}{R}\right)^{t-1}\left\{Y(k_t(\theta^t), q_t(\theta^t)) - C(k_t(\theta^t), \bar{q}_t) - M_t(r(\theta^t)) - T_t(\theta^t)\right\}\right\}$$

Partial problem $P(\bar{q})$:

Fix sequence of $\bar{q} \equiv \{\bar{q}_t\}_t$, solve screening problem subject to consistency of agents' choices with $\bar{q}$.

Full problem P:

P: $max_{\bar{q}} P(\bar{q})$.

Incentive Compatibility and a First-order Approach

- Expected continuation utility of firm after history θ^t:
  $V_t(\theta^t) = \sum_{t=s}^{T} (\frac{1}{R})^{t-s} \cdot \left\{ \int_{\Theta^t} \left\{ T_t(\theta^t) - \phi_t(I_t(\theta^t)) \right\} P(\theta^t | \theta^s) d\theta^t \right\}$

- Lifetime utility for a given sequence of realizations θ^∞: $\tilde{U}(\theta^\infty)$

- Envelope condition (Pavan, Segal and Toikka, 2014):
  $\frac{\partial V_t(\theta^t)}{\partial \theta_t} = \mathbb{E}\{\sum_{s=t}^{\infty} I_{t,s} \frac{\partial \tilde{U}(\theta^\infty)}{\partial \theta_s}\}$
  - $I_{t,s}$: impulse response of shock θ_t on time s shock θ_s. For AR(1) is p^{s-t}.
  - Relies on first-order condition (sufficiency?)

- $V_1(\theta_1) = V_1(\underline{\theta}_1) + \int_{\underline{\theta}_1}^{\theta_1} \frac{\partial V_1(\theta)}{\partial \theta} d\theta.$

- Expected PDV of transfers = expected PDV of disutility costs + info rent ($V_1(\theta_1)$).

Program: Virtual Surplus with Spillovers

If quantity can be controlled, set to maximize output net of production costs:

$$\tilde{Y}^*(q_t(\theta^t), \bar{q}_t) = \max_k \{ Y(k, q_t(\theta^t)) - C(k, \bar{q}_t) \}$$

$$P(\bar{q}) = \max W(\bar{q}) = \mathbb{E}\{\sum_{t=1}^{T} \left(\frac{1}{R}\right)^{t-1} \{ \tilde{Y}^*(q_t(\theta^t), \bar{q}_t) - M_t(r_t(\theta^t)) - \phi_t(l_t(\theta^t))$$

$$- V_1(\underline{\theta_1}) - \frac{1 - F^1(\theta_1)}{f^1(\theta_1)} I_{1,t} \frac{\partial \tilde{U}_t}{\partial \theta_t} \} \}$$

$$\text{s.t.:} \quad \int_{\Theta^t} q_t(\theta^t) P(\theta^t) d\theta^t = \bar{q}_t \qquad [\eta_t]$$

$$\text{and} \quad q_t(\theta^t) = q_{t-1}(\theta^{t-1})(1 - \delta) + \lambda_t(l_t(\theta^t), r_{t-1}(\theta^{t-1}), \theta_t)$$

Program: Virtual Surplus with Spillovers

If quantity can not be controlled, set by firm to maximize profits:

$$\tilde{Y}(q_t(\theta^t), \bar{q}_t) = \max_k \{p(k, q_t(\theta^t))k - C(k, \bar{q}_t)\}$$

$$P(\bar{q}) = \max W(\bar{q}) = \mathbb{E}\{\sum_{t=1}^{T} \left(\frac{1}{R}\right)^{t-1} \{\tilde{Y}(q_t(\theta^t), \bar{q}_t) - M_t(r_t(\theta^t)) - \phi_t(I_t(\theta^t)) - V_1(\underline{\theta_1}) - \frac{1 - F^1(\theta_1)}{f^1(\theta_1)} I_{1,t} \frac{\partial \tilde{U}_t}{\partial \theta_t}\}\}$$

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Wedges: Measures of Distortions in the Allocations

Akin to "implicit" taxes and subsidies.

$$\tau(\theta^t) \equiv \underbrace{\mathbb{E}\left(\sum_{s=t}^{\infty}\left(\frac{1-\delta}{R}\right)^{s-t}\frac{\partial \pi_s(q_s(\theta^s),\bar{q}_s)}{\partial q_s(\theta^s)}\frac{\partial \lambda_t(\theta^t)}{\partial l_t(\theta^t)}\right)}_{\text{Marginal benefit}} - \underbrace{\phi'(l_t(\theta^t))}_{\text{Marginal cost}}$$

$$s(\theta^t) \equiv \underbrace{M_t'(r(\theta^t))}_{\text{Marginal cost}} - \underbrace{\frac{1}{R}\mathbb{E}\left(\sum_{s=t+1}^{\infty}\left(\frac{1-\delta}{R}\right)^{s-t-1}\frac{\partial \pi_s(q_s(\theta^s),\bar{q}_s)}{\partial q_s(\theta^s)}\frac{\partial \lambda_{t+1}(\theta^{t+1})}{\partial r_t(\theta^t)}\right)}_{\text{Marginal benefit}}$$

Defined relative to laissez-faire with some profits $\pi_s(q_s(\theta^s), \bar{q}_s)$ (e.g.: patent protection).

Introduce Some Notation

$\Pi_t(\theta^t) \equiv \frac{1}{R}\left(\sum_{s=t}^{\infty}\left(\frac{1-\delta}{R}\right)^{s-t}\frac{\partial\pi(q(\theta^s),\bar{q}_s)}{\partial q_s}\right)$ (impact of q_t on profit stream).

$Q_t^*(\theta^t) \equiv \frac{1}{R}\left(\sum_{s=t}^{\infty}\left(\frac{1-\delta}{R}\right)^{s-t}\frac{\partial\tilde{Y}^*(q(\theta^s),\bar{q}_s)}{\partial q_s}\right)$ (impact on social surplus, if quantity controlled).

$Q_t(\theta^t) \equiv \frac{1}{R}\left(\sum_{s=t}^{\infty}\left(\frac{1-\delta}{R}\right)^{s-t}\frac{\partial\tilde{Y}(q(\theta^s),\bar{q}_s)}{\partial q_s}\right)$ (impact on social surplus, if quantity not controlled).

Optimal Profit wedge and R&D subsidy

$$\tau(\theta^t) = -\mathbb{E}\left(\sum_{s=t}^{\infty}(1-\delta)^{s-t}\eta_s\right)\frac{\partial\lambda_t(\theta^t)}{\partial l_t} - \mathbb{E}\left(Q_t^* - \Pi_t\right)\frac{\partial\lambda_t(\theta^t)}{\partial l_t}$$
$$+\frac{1-F^1(\theta_1)}{f^1(\theta_1)} l_{1,t}(\theta^t)\frac{\phi_t'\lambda_{\theta t}}{\lambda_t}\left[\frac{1}{\varepsilon_{l,1-\tau}}\frac{1}{\varepsilon_{\lambda l,t}} + \rho_{\theta l,t}\right]$$

$$s(\theta^t) = \mathbb{E}\left(\sum_{s=t+1}^{\infty}(1-\delta)^{s-t-1}\eta_s\frac{\partial\lambda(\theta^{t+1})}{\partial r_t}\right) + \mathbb{E}\left(\left(Q_{t+1}^* - \Pi_{t+1}\right)\frac{\partial\lambda(\theta^{t+1})}{\partial r_t}\right)$$
$$+\frac{1}{R}\mathbb{E}\left(\frac{1-F^1(\theta_1)}{f^1(\theta_1)} l_{1,t+1}(\theta^{t+1})\phi_{t+1}'(l(\theta^{t+1}))\frac{\lambda_\theta\lambda_r}{\lambda\lambda_l}(\rho_{lr} - \rho_{\theta r})\right)$$

Optimal Profit wedge and R&D subsidy

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Pigouvian Correction:

If positive externality, subsidize profits and R&D.

Larger for high productivity firms as long as $\rho_{\theta l} > 0$ and $\rho_{\theta r} > 0$.

Optimal Profit wedge and R&D subsidy

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Screening:

Stochastic productivity process:

Lower persistence: lower wedges over time.

Special cases: iid, full persistence, AR(1).

Larger inverse hazard ratio: larger wedges (no distortion at the top).

Optimal Profit wedge and R&D subsidy

$$\tau(\theta^t) = -\mathbb{E}\left(\sum_{s=t}^{\infty}(1-\delta)^{s-t}\eta_s\right)\frac{\partial\lambda_t(\theta^t)}{\partial l_t} - \mathbb{E}\left(Q_t^* - \Pi_t\right)\frac{\partial\lambda_t(\theta^t)}{\partial l_t} + \frac{1-F^1(\theta_1)}{f^1(\theta_1)} l_{1,t}(\theta^t)\frac{\phi_t'\lambda_{\theta t}}{\lambda_t}\left[\frac{1}{\varepsilon_{l,1-\tau}}\frac{1}{\varepsilon_{\lambda l,t}} + \rho_{\theta l,t}\right]$$

$$s(\theta^t) = \mathbb{E}\left(\sum_{s=t+1}^{\infty}(1-\delta)^{s-t-1}\eta_s\frac{\partial\lambda(\theta^{t+1})}{\partial r_t}\right) + \mathbb{E}\left(\left(Q_{t+1}^* - \Pi_{t+1}\right)\frac{\partial\lambda(\theta^{t+1})}{\partial r_t}\right) + \frac{1}{R}\mathbb{E}\left(\frac{1-F^1(\theta_1)}{f^1(\theta_1)} l_{1,t+1}(\theta^{t+1})\phi_{t+1}'(l(\theta^{t+1}))\frac{\lambda_\theta\lambda_r}{\lambda\lambda_l}(\rho_{lr} - \rho_{\theta r})\right)$$

Screening:

Efficiency cost of distorting R&D effort:

Allocative efficiency: inverse elasticity rule.

Informational rent: increasing in complementarity effort-type $\rightarrow$ less costly mimicking of low types.

Optimal Profit wedge and R&D subsidy

$$\tau(\theta^t) = -\mathbb{E}\left(\sum_{s=t}^{\infty}(1-\delta)^{s-t}\eta_s\right)\frac{\partial\lambda_t(\theta^t)}{\partial l_t} - \mathbb{E}\left(Q_t^* - \Pi_t\right)\frac{\partial\lambda_t(\theta^t)}{\partial l_t} + \frac{1-F^1(\theta_1)}{f^1(\theta_1)}l_{1,t}(\theta^t)\frac{\phi_t'\lambda_{\theta t}}{\lambda_t}\left[\frac{1}{\varepsilon_{l,1-\tau}}\frac{1}{\varepsilon_{\lambda l,t}} + \rho_{\theta l,t}\right]$$

$$s(\theta^t) = \mathbb{E}\left(\sum_{s=t+1}^{\infty}(1-\delta)^{s-t-1}\eta_s\frac{\partial\lambda(\theta^{t+1})}{\partial r_t}\right) + \mathbb{E}\left(\left(Q_{t+1}^* - \Pi_{t+1}\right)\frac{\partial\lambda(\theta^{t+1})}{\partial r_t}\right) + \frac{1}{R}\mathbb{E}\left(\frac{1-F^1(\theta_1)}{f^1(\theta_1)}l_{1,t+1}(\theta^{t+1})\phi_{t+1}'(l(\theta^{t+1}))\frac{\lambda_\theta\lambda_r}{\lambda\lambda_l}(\rho_{lr} - \rho_{\theta r})\right)$$

Screening:

Efficiency cost of distorting R&D investments:

Higher $\rho_{lr} \rightarrow$ larger s. Incentivizes unobservable input, relaxes IC.

Higher $\rho_{\theta r} \rightarrow$ smaller s. Increases info rent, tightens IC.

Special case: $\rho_{lr} = \rho_{\theta r}$. Only distort R&D if improves screening and incentives for unobservable input.

Optimal Profit wedge and R&D subsidy

$$\tau(\theta^t) = -\mathbb{E}\left(\sum_{s=t}^{\infty}(1-\delta)^{s-t}\eta_s\right)\frac{\partial\lambda_t(\theta^t)}{\partial I_t} - \mathbb{E}\left(Q_t^* - \Pi_t\right)\frac{\partial\lambda_t(\theta^t)}{\partial I_t}$$

$$+ \frac{1-F^1(\theta_1)}{f^1(\theta_1)} I_{1,t}(\theta^t)\frac{\phi_t'\lambda_{\theta t}}{\lambda_t}\left[\frac{1}{\varepsilon_{I,1-\tau}}\frac{1}{\varepsilon_{\lambda I,t}} + \rho_{\theta I,t}\right]$$

$$s(\theta^t) = \mathbb{E}\left(\sum_{s=t+1}^{\infty}(1-\delta)^{s-t-1}\eta_s\frac{\partial\lambda(\theta^{t+1})}{\partial r_t}\right) + \mathbb{E}\left(\left(Q_{t+1}^* - \Pi_{t+1}\right)\frac{\partial\lambda(\theta^{t+1})}{\partial r_t}\right)$$

$$+ \frac{1}{R}\mathbb{E}\left(\frac{1-F^1(\theta_1)}{f^1(\theta_1)} I_{1,t+1}(\theta^{t+1})\phi_{t+1}'(I(\theta^{t+1}))\frac{\lambda_\theta\lambda_r}{\lambda\lambda_I}(\rho_{Ir} - \rho_{\theta r})\right)$$

Monopoly Quality Valuation Correction:

Wedges defined relative to patent system: monopolist does not value quality as much as society, needs extra incentive to invest.

Disappears if wedges defined relative to prize system: social and private valuations aligned.

Optimal R&D policy depends on IPR.

When Quantity Cannot be Controlled

Imagine irremovable patent system $\rightarrow$ monopoly quantity $k_t(q_t(\theta^t), \bar{q}_t)$ chosen for any quality.

Same formulas, but Q_t replaces Q_t^*.

Lesson: Optimal R&D policy depends on IPP.

Improving quality through R&D effort and investment subsidies here generates extra benefit by increasing monopolist's quantity.

$$\frac{\partial \tilde{Y}(q_t(\theta^t), \bar{q}_t)}{\partial q} = \underbrace{\frac{\partial Y(q_t(\theta^t), k_t(q_t(\theta^t), \bar{q}_t))}{\partial q_t(\theta^t)}}_{\text{Direct benefit}} + \underbrace{\left(p(q_t(\theta^t), k_t(q_t(\theta^t), \bar{q}_t)) - \frac{\partial C}{\partial k} \right)}_{\text{Monopoly distortion}} \frac{\partial k_t(q_t(\theta^t), \bar{q}_t)}{\partial q_t(\theta^t)}$$

Larger subsidy, lower tax, but lower investments overall, at higher cost (additional costly constraint).

Extensions

(1) **Different types of R&D investments:**

$$\lambda_t = \lambda_t(r^1_{t-1}, ... r^j_{t-1}, ..., r^J_{t-1}, l_t, \theta_t)$$

$s^j(\theta^t)$ depends on i) externality $\frac{\partial \lambda_t}{\partial r^j_{t-1}}$, ii) complementarity: $\rho^j_{\theta l} - \rho^j_{\theta r}$.

→ subsidize investments with higher externalities, but less so if they are highly complementary with unobservable firm productivity.

(2) **Different externalities:**

$$C(k, \bar{q}^1_t, .., \bar{q}^J_t) \quad \text{with} \quad \bar{q}^j_t = \int_{\Theta^t} q^j_t(\theta^t) d\theta^t$$

$$\text{and} \quad q^j_t(\theta^t) = q^j_t(\theta^{t-1})(1-\delta) + \lambda^j_t(r^j_{t-1}, l_t, \theta_t)$$

Basic vs. Applied research?

Implementation Results

Many possible (theoretically equivalent) implementations.
Administrative/political constraints may matter in practice.

Optimal allocation when quantity can be controlled can be implemented:

1) with price subsidy ($p(k,q)(1+s_p(p,k)) = \frac{Y(k,q)}{k}$) plus age-dependent tax function $T_t(q_t, r_t, q_{t-1}, r_{t-1}, q_1)$.

with constant markup $Y(k,q) = \frac{1}{1-\beta} q^{\beta} k^{1-\beta}$, constant $s_p = \frac{\beta}{1-\beta}$.

2) with prize $G_t(\lambda_t, q_{t-1}, r_t, r_{t-1}, q_1)$, government purchases innovation from firms, produces the socially optimal quantity.

Allocation when quantity can not be controlled implemented by tax $T_t^n(q_t, r_t, q_{t-1}, r_{t-1}, q_1)$ (no price subsidy).

Quantitative Investigation

Dataset Information: Compustat/LBD and Patent Data

Patent data from USPTO matched to Compustat or LBD data.

For Compustat: Select firms as in Bloom, Schankerman and Van Reenen (2013):
patent $\geq$ once since 1963, observed $\geq$ 4 times in 1980–2001.

Variable	Mean	Median
Sales (in mil. USD)	3133	494
Citations per patent	7.7	6
Patents per year	18.5	1
R&D spending / sales	0.043	0.014
Number of employees (000's)	18.4	3.8
Number of firms	736	

λ = flow of citations per patent. q = depreciated stock.

Functional Forms for Estimation

Function	Notation	Functional form
Consumer valuation	$Y(q_t, k_t)$	$\frac{1}{1-\beta} q_t^{\beta} k_t^{1-\beta}$
Cost function	$C_t(k, \bar{q}_t)$	$\frac{k}{\bar{q}_t^{\zeta}}$
Quality accumulation	$H(q_{t-1}, \lambda_t)$	$q_t = (1-\delta) q_{t-1} + \lambda_t$
Step size	$\lambda_t(r_{t-1}, l_t, \theta_t)$	$(\alpha r_{t-1}^{1-\rho_{\theta r}} + (1-\alpha)\theta_t^{1-\rho_{\theta r}})^{\frac{1}{1-\rho_{\theta r}}} l_t$
Disutility of effort	$\phi_t(l_t)$	$\kappa_l \frac{l_t^{1+\gamma}}{1+\gamma}$
Cost of R&D	$M_t(r_t)$	$\kappa_r \frac{r_t^{1+\eta}}{1+\eta}$
Stochastic type process	$f^t(\theta_t \vert \theta_{t-1})$	$\log \theta_t = p \log \theta_{t-1} + (1-p)\mu_\theta + \epsilon_t$
Distribution of heterogeneity θ_1	$f^1(\theta_1)$	$f^1(\theta_1) = \frac{I_{\Theta 1}(\theta_1)}{\theta_1 [\underline{\theta}_1 - \bar{\theta}_1]}$
Initial quality level	q_0	0

Estimation Targets: Moments

Moment	Target	Simulation
M1. Patent quality-R&D elasticity	0.879	0.956
M2. R&D/Sales median	0.041	0.029
M3. Sales growth (DHS) mean	0.060	0.072
M4. Within-firm patent quality coeff of var	0.630	0.720
M5. Across-firm patent quality coeff of var (young)	1.055	1.026
M6. Across-firm patent quality coeff of var (old)	0.991	0.768
M7. Patent quality young/old	1.043	1.022
M8. Spillover regression coefficient	0.191	0.190
M9. Subsidy regression coefficient	0.350	0.362

Parameters to be estimated: $\chi = (\alpha, \rho_{\theta r}, \sigma_{\varepsilon}, p, \kappa_l, \kappa_r, \gamma, \zeta, \Theta^1)$

Loss function: $L(\chi) = \sum_{k=1}^{9} \left(\frac{\text{moment}_k^{\text{model}}(\chi) - \text{moment}_k^{\text{data}}}{\text{moment}_k^{\text{data}}} \right)^2$

Estimation Targets: Moments

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M8. Spillover regression coefficient	0.191	0.190
M9. Subsidy regression coefficient	0.350	0.362

Replicate Bloom, Schankerman, and Van Reenen (2013) IV estimates.

Randomly draw κ_r in $M(r) = \kappa_r \frac{r_t^{1+\eta}}{1+\eta}$.

Match regression coefficient of q_t on average R&D stock.

Estimation Targets: Moments

Moment	Target	Simulation
M1. Patent quality-R&D elasticity	0.879	0.956
M2. R&D/Sales median	0.041	0.029
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M7. Patent quality young/old	1.043	1.022
M8. Spillover regression coefficient	0.191	0.190
M9. Subsidy regression coefficient	0.350	0.362

Replicate Bloom et al. effects of R&D tax credits estimates.

Randomly draw κ_r in $M(r) = \kappa_r \frac{r_t^{1+\eta}}{1+\eta}$.

Match regression coefficient R&D stock on cost of R&D (tax credits).

Estimated Parameters

Parameter	Symbol	Value
Interest rate	R	1.050
R&D share	α	0.398
Knowledge share	β	0.150
Intangibles depreciation	δ	0.100
Type variance	σ_{ϵ}	0.337
R&D cost elasticity	η	1.500
Effort cost elasticity	γ	1.095
Scale of disutility	κ_l	0.771
Scale of R&D cost	κ_r	0.050
Support for θ_1	Θ^1	2.025
Leve of types	μ_{θ}	0.000
Type persistence	p	0.629
Initial intangibles	q_0	0.000
Initial R&D stock	r_0	1.000
R&D-type substitution	$\rho_{\theta r}$	1.451
Production externality	ζ	0.033

Gross and Net Incentives

Gross subsidy vs. net subsidy (on top of making R&D expenses corporate tax-deductible).

Gross subsidy $\tilde{s}$:

$$\pi(1-\tau) - (1-\tilde{s})M(r)$$

Net incentive for R&D is s such that:

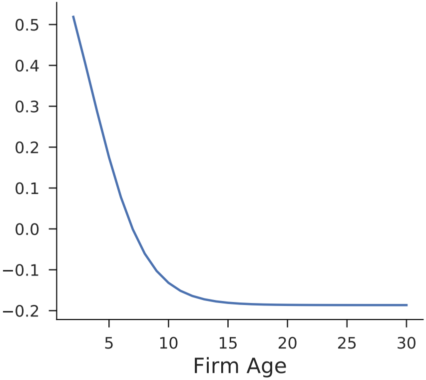
$$\underbrace{(\pi - M(r))}_{\text{Deduct R\&D expenses}} (1-\tau) - \underbrace{(1-s)}_{\text{Net subsidy}} M(r)$$

Relation: $s = \tilde{s} - \tau$.

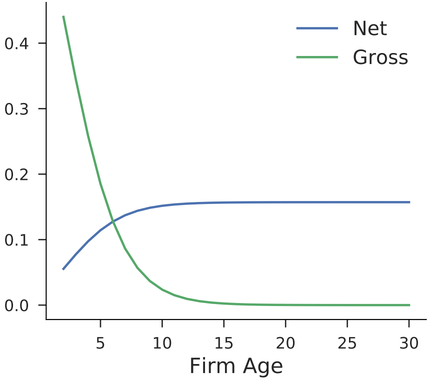
Same idea for wedges.

Young vs Old Firms

(a) Profit wedge

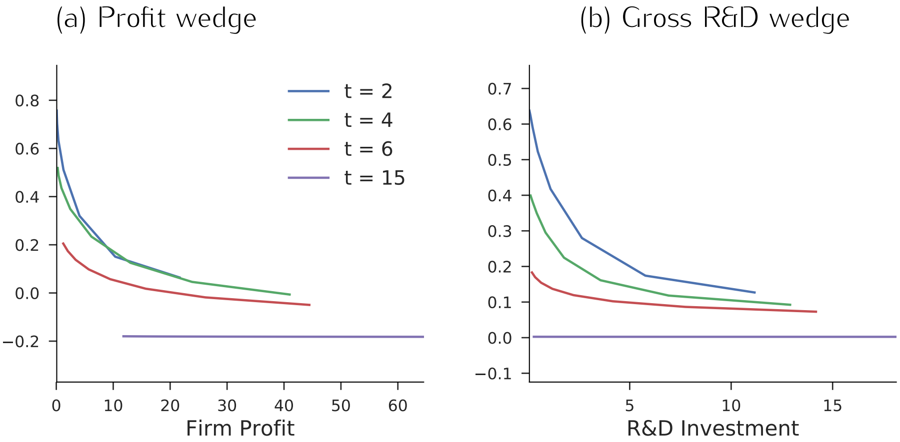


(b) Gross & Net R&D wedges



Policies converge to Pigouvian correction. Screening term for R&D ≤ 0 since $\rho_{\theta r} > \rho_{Ir} = 1$ (net wedge $\uparrow$)

Wedges as Function of Profits and R&D Expenses



Marginal tax rate and R&D subsidy lower for higher productivity firms.

"No distortion at the top" ▸ Comparative Statics ▸ No IPR ▸ Allocations

Optimal Simpler Policies

How Close can Simpler Policies Come?

Linear Policies

$$T(\pi) = \tau_0 \pi \qquad S(M) = s_0 M$$

Linear Policies with Interaction Term

$$T(\pi, M) = (\tau_0 + \tau_1 M)\pi \qquad S(M) = s_0 M$$

Heathcote-Storesletten-Violante (HSV) Policies

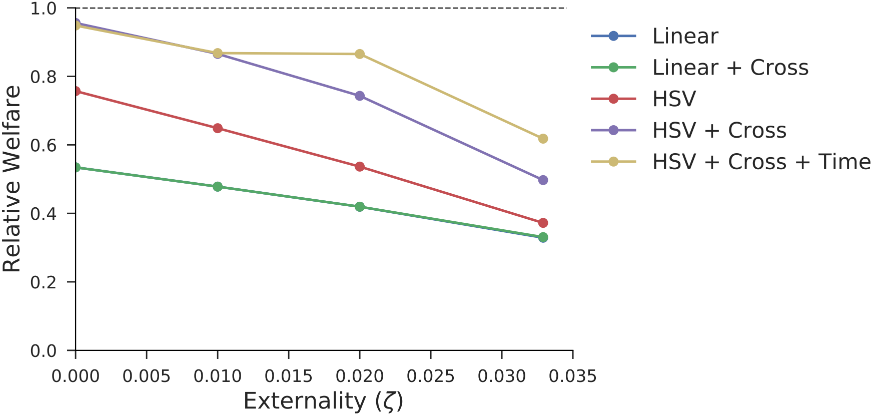
$$T(\pi) = \tau_0 \pi - \tau_1 1 + \tau_2 \pi^{1+\tau_2} \qquad S(M) = s_0 M - s_1 1 + s_2 M^{1+s_2}$$

HSV Policies with Interaction Term

$$T(\pi, M) = (\tau_0 + \tau_3 M^{s_2})\pi - \tau_1 1 + \tau_2 \pi^{1+\tau_2}$$

$$S(M) = s_0 M - s_1 1 + s_2 M^{1+s_2}$$

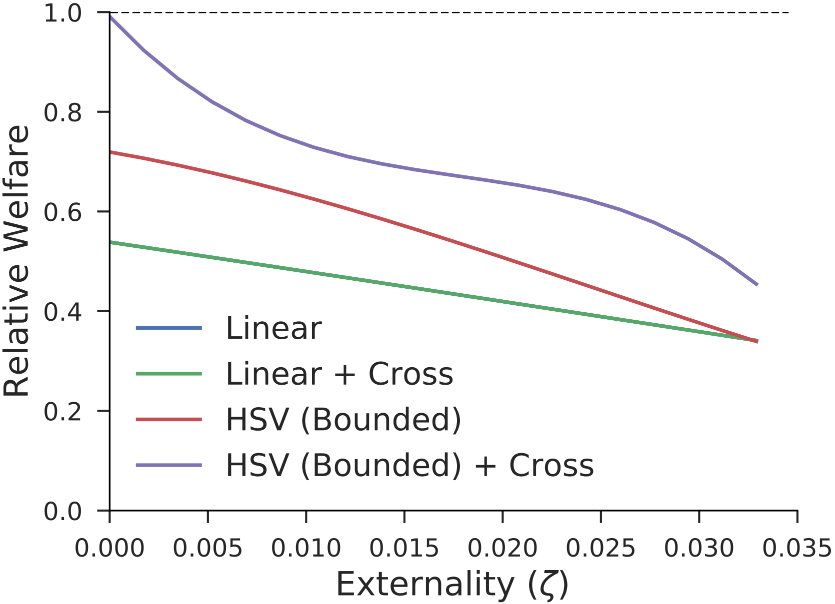
Simpler Policies: Revenue Gains



Optimal Simpler Policies

Externality	Optimal Policy							Revenue (% of optimum)
	A. Linear Policies							
	$T(\pi) = \tau_0 \pi \qquad S(M) = s_0 M$							
$\zeta = 0.033$	$\tau_0 = 0.40$	$s_0 = 0.35$						33%
	B. Linear Policies with Interaction Term							
	$T(\pi, M) = (\tau_0 + \tau_1 M)\pi \qquad S(M) = s_0 M$							
	C. Heathcote-Storesletten-Violante (HSV) Policies							
	$T(\pi) = \tau_0 \pi - \frac{\tau_1}{1+\tau_2} \pi^{1+\tau_2} \qquad S(M) = s_0 M - \frac{s_1}{1+s_2} M^{1+s_2}$							
$\zeta = 0.033$	$\tau_0 = 0.71$	$\tau_1 = 0.18$	$\tau_2 = 0.18$	$s_0 = 0.58$	$s_1 = 0.25$	$s_2 = 0.40$		37.2%
	D. HSV Policies with Interaction Term							
	$T(\pi, M) = (\tau_0 + \tau_3 M^{s_2})\pi - \frac{\tau_1}{1+\tau_2} \pi^{1+\tau_2} \qquad S(M) = s_0 M - \frac{s_1}{1+s_2} M^{1+s_2}$							
$\zeta = 0.033$	$\tau_0 = 0.48$	$\tau_1 = 0.24$	$\tau_2 = 0.35$	$\tau_3 = 0.40$	$s_0 = 0.51$	$s_1 = 0.17$	$s_2 = 0.42$	49.8%

Simpler Policies with Patent System as given: Revenue Gains



Conclusion

- Model of innovation with heterogeneous firms, private information, and spillovers.
  - Use mechanism design to solve for constrained efficient allocations.
  - Implementation by a tax/subsidy or prize mechanism.
- Externality $\rightarrow$ Optimal to subsidize R&D investments.
- Asymmetric information could go other way in theory if R&D very complementary to firm productivity ($\uparrow$ informational rents to firms).
- Revenue loss from restricted policies is large, but an HSV policy with interaction term between R&D and profits comes close.